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THE CONSTRUCTION OF MORTALITY AND SICKNESS TABLES

A PRIMER

BY

W. PALIN ELDERTON

AND

RICHARD C. FIPPARD

FELLOWS OF THE INSTITUTE OF ACTUARIES

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PREFACE TO FIRST EDITION

In the following pages we have attempted to describe, with as little technicality as possible, the usual methods of constructing mortality and similar tables.

As this book is intended mainly for those who have little or no acquaintance with the subject, we have tried to explain each step verbally with the help of arithmetical examples, in order to avoid the introduction of algebraical formulæ. It seems to us that such formulæ are of little use in explaining the principles of the subject to beginners.

We have tried to avoid debatable points, but we feel that in some directions the subject has not yet been advanced beyond the empirical stage. This is particularly true of the treatment of census data. Much work has been done in perfecting the details of the census methods, but their bases rest on very weak foundations, and it seems that until fuller and more accurate information is available the value of the assumptions at present made cannot be properly determined.

In conclusion, we desire to record our indebtedness to our friend J. A. Humphreys for many valuable suggestions. He most kindly offered to read the book again in proof, but his sudden death has deprived us of this further help.

W. P. E.
R. C. F.

FEBRUARY 1914.

PREFACE TO SECOND EDITION

I WISH to thank one or two friends, and particularly A. Henry, for suggestions in connection with the new Edition of our book. I have inserted in Chapter V a note on marriage rates, altered and enlarged Chapter VII, and made a small amount of textual variation elsewhere.

The preparation of a new Edition of our book has been a sad reminder of the death of R. C. Fippard, who was killed in Gallipoli in June 1915. I think he would have approved of the alterations.

W. P. E.

NOTE

FOR the Third Edition, Chapter IX has been rewritten, and a few alterations have been made elsewhere.

W. P. E.

JANUARY 1935.

CONTENTS

THE CONSTRUCTION OF MORTALITY AND SICKNESS TABLES

CHAPTER I

INTRODUCTORY

It is well known that the practical conclusions of an actuary are based on mortality, sickness, and other tables, which are prepared from the statistics given in the census returns or collected by life assurance offices and friendly societies.

The study of the methods by which these tables are constructed is therefore of great importance, and it would naturally be anticipated that the fundamental character of the subject would make it attractive; but, as a matter of fact, there is no other part of actuarial work that appears to the average student so troublesome and uninteresting. This may be because he learns about several tables of mortality which are never used, some of which never have been used, and many of which were constructed by more or less unsuitable methods, and he is left with the erroneous impression that the whole subject is a mass of bewildering detail. He may even go so far as to think that he would employ his time more profitably by evolving methods to suit special circum-

stances, than in learning the way other people have solved, or failed to solve, similar problems. And here he is right. Thinking out such solutions is the best way to understand the subject; and when it has been mastered the history is easy enough and less uninteresting.

The real difficulty for most people is that the amount of detail obscures not only the importance of the subject but even the problem that has to be solved. This is, of course, fatal; when we are trying to solve any problem we must be clear about its nature, and if, as is nearly always the case in statistical work, an approximate result is all that can be hoped for, we must try to see where the approximation falls short of accuracy.

Let us begin then by saying that the object of our investigation is to find the rate of mortality at any age from data obtained from censuses and death registers of the general population, or from the books of insurance offices, where the "rate of mortality" at a specified age may be defined as the ratio of the number of persons in a particular population dying within one year from the attainment of that age, to the number who were under observation for one year from the attainment of that age, or until death if occurring within the year. If, for example, there are 10,000 people aged 30 exactly, each of whom is kept in sight until he attains age 31 or dies (if death occurs before that age), and if it is found that there are 104 deaths among the 10,000, then the rate of mortality, or chance of dying in a year at age 30, is ·0104.

Now let us see how this definition can help us to appreciate some of the difficulties of our problem and the ways these difficulties can be met. In the first place, we want a large number of people (in our numerical illustration, 10,000) who are exactly 30 years of age. We cannot expect to trace many people who were all born on the same day, but we might trace 10,000 people all of whom attained age 30 in the same year (or some other period) and observe them from their thirtieth to their thirty-first birthdays, or we might observe for a year all those who are between 29½ and 30½ at a particular moment. In the second place, we must know the number of deaths among the people observed; and in the third place, our definition suggests that we must observe each case for a year. Here we are faced with the difficulty that people emigrate or, if we are dealing with particulars from the registers of an assurance office, some policies lapse or are surrendered and the lives are consequently lost sight of. If there were 10,200 people all aged 30, of whom 400 withdrew (by emigration or their policies lapsing) at the end of exactly half a year, we ought to consider the 400 people withdrawing as equivalent to 200 under observation throughout the year, because we do not know whether they died in the second half of the year or not; in other words, within our experience these 400 who withdrew only had half the chance of dying in the year that was given to the rest of the lives under observation. The consequence is that the 10,200 people in the circumstances described are equivalent to 10,000

people observed for the whole year. If the 400 left at various times during the year a similar argument will hold, and we shall see later how these points are dealt with in practice.

As we have already indicated, we can use either the data obtained from the general population, or the particulars that can be found in the registers of a life assurance company, to reach the rates of mortality at various ages; but more accurate results can be obtained from the latter, owing to the more detailed information available. We shall therefore deal with the life office methods first, and refer afterwards to the methods to be adopted in the case of the general population.

The course that is usually followed in order to obtain rates of mortality, is first to decide exactly how many years of the assurance company's experience are to be taken into account and whether any cases are to be excluded. It might, for instance, be thought well to exclude those who effected policies many years ago or those who were charged an extra premium, while for some purposes the separation of with-profit policies from without-profit policies might be deemed advisable. Males and females should be dealt with separately whenever possible. If it is decided to neglect certain cases, great care must be taken to see that their exclusion will not vitiate the results of the investigation. Thus deaths arising from some specified cause, such as cancer, could not be excluded, because each life has undergone the risk of dying from this disease, and if we exclude those persons whose deaths have actually taken place from

this cause the general rate of mortality would be under-estimated. There would, however, be no fallacy

Life Assured . . *ADAM SMITH*

Policy Number . *1001*

Date of Exit	*30 March 1886*
Date of Entry	*30 Dec. 1879*
Duration	*6*
Date of Birth	*1 Jan. 1850*
Age at Entry	*30*
Mode of Exit	*Lapse*

REMARKS

in working only on people resident in a certain district or following a certain occupation.

Having decided on these points, the usual practice

is to fill up a card (see specimen) for every case to be included, giving the date of birth, date of entry, the date of withdrawal and the mode of withdrawal, *i.e.* death, lapse, surrender, or if the life was still living at the close of the observations.

The above specimen card is suitable, the corner is cut off for convenience in sorting, and the particulars are arranged so that the calculations can be made readily.[1]

As the easiest way to appreciate what these particulars give and how they can be used, is by taking one or two examples, Table I has been prepared so that the first four lines give the particulars furnished by the office, and the later lines the calculated ages and durations to be used in the subsequent work.

The particulars in the first four lines are easy to follow, and can be left to explain themselves while we consider the rest of the table.

It will be seen in the first place that the nearest age at entry has been calculated in each case. This is used as a convenient and accurate approximation to the exact age; it means that we group together all persons on whose lives policies have been granted between the ages $29\frac{1}{2}$ and $30\frac{1}{2}$, and assume the exact age at entry to have been 30.

The next two lines of the table show the method of calculating the time during which a policy is in

[1] If the experience extends to a very large number of cases, mechanical means of sorting would probably be used. Cards are printed with a series of columns in which holes can be punched, and these holes enable the sorting machine to group and enumerate the cards.

force in various circumstances.* Taking the first example, we see that the duration from the date of entry to the day when the policy lapsed was six years and three months. The life assured comes under our

TABLE I

	EXAMPLE 1.	EXAMPLE 2.	EXAMPLE 3.	EXAMPLE 4.	EXAMPLE 5.
Date of birth .	Jan. 1, 1850	June 23, 1851	Dec. 15, 1860	March 10, 1845	Sept. 15, 1850
Date of entry .	Dec. 30, 1879	Jan. 1, 1880	Jan. 5, 1880	June 10, 1880	June 18, 1880
Date of withdrawal	March 30, 1886	Aug. 4, 1900	Dec. 10, 1895	Oct. 15, 1907	...
Mode of withdrawal	Lapse	Surrender	Death	Death	Existing on June 18, 1911
Nearest age at entry	30	29	19	35	30
Duration . .	$6\frac{3}{12}$	$20\frac{7}{12}$	$15\frac{11}{12}$	$27\frac{4}{12}$	31
Duration used in calculations	6	21	15	27	31
Age at exit used in calculations	36	50	34	62	61

observation for that time, and no longer; he might, within our experience, have died during that period, but as a matter of fact he did not do so; or in more technical language, he was exposed to risk of death for six years and three months. In the same way, the life assured in the second case was exposed to risk of death for twenty years and seven months. If we go back to our definition of the rate of mortality, we see that these cases do not fit in conveniently, because we ought to observe every case until the end of the year. The first six years in the first example fit well enough, but the life assured had a chance of dying in only three months of the seventh. We ought, strictly speaking, to count him for one quarter of that year, and the assured in the second example for twenty years and seven-twelfths of the twenty-first year. In practice it is convenient to avoid fractional durations, so we take the nearest duration in a similar way to that followed in connection with the age at entry. We assume, therefore, that Example 1 was exposed to risk for six years, and Example 2 for twenty-one years. We balance the understatement of some cases with the overstatement of others.

The next two examples relate to deaths, and have been dealt with differently from those that have just been considered. We have not taken the nearest duration but the number of complete years lived, and neglected the odd months altogether. To understand this we must go back again to our definition of the rate of mortality. In finding the rate of mortality at age 30, for instance, we want to obtain the ratio of the deaths between ages 30 and 31 (which in

practice we call the deaths at age 30) to the corresponding number living at age 30, so that if a person dies at any time between 30 and 31 we must assume he was exposed to risk for the whole of that year of age. What we have to find is the chance of a man dying in a year, and it does not matter in what portion of the year death occurs. If we adopted any other method, such as the nearest duration, we should reach some other function than the rate of mortality as we have defined it.

The one remaining example relates to a policy which is still in existence when the observations end. The usual practice is to assume that the experience ends not on a fixed date, but on the anniversaries in a particular calendar year (*e.g.* 1911) of the dates when the policies were effected. This fits in well with our definition of the rate of mortality, because it enables us to observe these cases for an exact number of years.

Those who are studying the subject of the construction of mortality tables for the first time frequently have considerable difficulty in following the meaning of the term "those existing at the close of observations," and seem to find it hard to see why they have to be taken into account. It must be borne in mind that the experience covers a fixed period of years. Each policy is observed from the date of entry until its cessation by surrender, lapse, or death, or until its anniversary in the final year of the period. There will necessarily be several of the latter cases, some effected one year before the final year, some two years, and so on. These are termed

the "Existing at the close of Observations," and since the policy-holders might have died during the time covered by the experience, they must be included for each year of their existence up to the date when the observations end.

This may be made clearer if we consider four policies effected ten years ago. Let us assume that in one case the life assured died; in another the policy was allowed to lapse; a third was surrendered; and the remaining one is still being maintained. This last one would be termed "Existing." It must be taken into account for each of the ten years, because the life assured might have died during that period, even though he did not do so. If it was excluded altogether we should exaggerate the mortality, as can easily be seen by considering the extreme case of ten persons observed for ten years, of whom only one died and none withdrew. If we did not take the nine existing people into account for the whole of the ten years, we should assume that a person was certain to die within that period, whereas as a matter of fact only one out of ten had done so.

Having thus provided ourselves with the necessary data, we have to consider how it can best be manipulated, but before doing so it will be useful to recapitulate what has been said in the following way, which shows the procedure to be adopted:—

1. Decide the limits of the experience.
2. Write a card for each case, giving—(*a*) date of birth; (*b*) date of entry; (*c*) date of exit; (*d*) cause of exit, or if existing.

3. Calculate the nearest age at entry in each case.
4. Calculate the nearest duration for withdrawals.
5. Calculate the curtate duration (*i.e.* integral number of years) for deaths.
6. Calculate the exact duration for those existing at the close of the observations by taking them up to the anniversaries of the policies in a chosen calendar year. In this connection a word of caution must be added. If the calendar year which ends the period is 1911 and the anniversary of the policy is 30th June, the policy passes out of observation on that date. This means that if the life assured died, or if the policy was surrendered between the 30th June and the end of the year, we should count the case as existing, and not as a death or surrender.

CHAPTER II

SELECT TABLES

THE mortality amongst assured lives depends not only upon the attained ages of the lives but on the ages at which they were examined or selected for life assurance. People who have just been accepted for life assurance are far less likely to die within one year than those of the same age accepted twenty years ago, consequently it is best for most purposes to tabulate the cases according to age at entry. The natural procedure, therefore, is to sort all the cards according to the age at entry and count up the number of entrants at each age. Table II gives the particulars for age 30 as they are usually scheduled for each age at entry.

In the experience from which this table was formed the number of cards in the bundle relating to age 30 was 1499. These cards were sorted into three groups—(1) existing, (2) withdrawals (*i.e.* lapses and surrenders), (3) deaths. Then each one of these groups was sorted according to the duration entered on each card, which had of course been previously calculated by the methods already described. The number of cards for each duration was then inserted in the table.

We can now go through the table and see what the entries mean. Opposite duration 0 are entered

all the cards in which that duration appears on the cards in accordance with the practice described on p. 11; thus there were 30 withdrawals; this tells us that 30 people allowed their policies to be discontinued within six months of the date of issue, the nearest duration in these cases being 0. Six people died within a year of the time when their policies

TABLE II

Nearest Age at Date of Assurance, 30.					
Duration.	Number of Entrants, 1499.				
	Existing at Close of Observations.	Withdrawals.	Deaths.	Exposed to Risk.	Rate of Mortality.
0	...	30	6	1469	0·00408
1	45	157	10	1261	0·00793
2	27	109	7	1115	0·00628
3	35	60	11	1013	0·01086
4	30	47	11	925	0·01189
5	48	34	6	832	0·00721
6	42	32	5	752	0·00665
7	25	26	9	696	0·01293
8	28	25	6	634	0·00946
etc.	etc.	etc.	etc.	etc.	etc.

were effected. There were no existing, because, as we observe up to policy anniversaries in a particular year, a case in order to be entered as existing must have been in force for at least a year. The exposed to risk is the equivalent of the "number of people observed for one year or until death" in our definition, *i.e.* the number of entrants making proper allowance for withdrawals and existing. The exposed to risk for dura-

tion 0 was 1469, *i.e.* 1499 entrants less the 30 withdrawals. The rate of mortality is $6 \div 1469 = \cdot 00408$.

Considering duration 1, we find that 45 people who were still living and whose policies were still in force when the observations closed, had taken out their policies a year before; 157 people had withdrawn at various times between six months and eighteen months (the nearest duration was one year); and 10 died between one and two years from the time when their policies were issued. Now since the 45 existing were observed up to their exact durations, none of them had a chance of dying in the second year of assurance 1–2, so they must be deducted in finding the exposed to risk for that year, and for similar reasons the 157 who withdrew must be deducted. We must also deduct the 6 people who died in the year 0–1; unless we do this we shall be giving them the chance of dying twice! The exposed to risk for duration 1 is therefore $1469 - 45 - 157 - 6 = 1261$, and the rate of mortality is $10 \div 1261 = \cdot 00793$. Subsequent durations follow in the same way, until all the 1499 cards are accounted for.

Similar tables are made for all other ages at entry, and finally the results can be collected in the form shown in Table III, which, if completed, would give the rates of mortality for all ages at entry, and all durations. Such tables are called "Select Tables of Mortality," because they show the mortality for each year subsequent to the date on which the life was "selected" for assurance by Medical Examination or otherwise.

This supplies one solution of the problem of finding

the rates of mortality from the books of insurance offices, but before turning to other methods it will be well to see where this method is open to criticism. The main points at which this can be directed are the use of the nearest age at entry and the nearest duration for withdrawals. The error in the former arises mainly because there is a tendency for people

TABLE III.—RATES OF MORTALITY (SELECT)

Age at Entry.	Duration.					
	0	1	2	3	4	Etc.
...	...	...	...	...	...	...
...	...	...	...	...	...	...
...	...	...	...	...	...	...
25	0·00281	0·00463	0·00532	0·00576	0·00614	etc.
26	0·00286	0·00469	0 00540	0·00585	0·00625	...
27	0·00292	0·00476	0·00549	0·00595	0·00637	...
28	0·00298	0·00484	0·00558	0·00607	0·00650	...
29	0·00305	0·00493	0·00569	0·00619	0·00664	...
30	0·00312	0·00502	0·00580	0 00633	0·00680	...
31	0·00321	0·00512	0·00592	0·00647	0·00698	...
32	0·00329	0·00523	0·00606	0·00664	0·00717	etc.
...	...	...	...	...	...	...
...	...	...	...	...	...	...
...	...	...	...	...	...	...
...	...	...	...	...	...	...

to enter before a birthday rather than after it, so that they may obtain the assurance at a lower rate of premium, with the result that the true average age of each group is slightly less than the assumed age, and the rates of mortality for each year of assurance relate to slightly lower ages at entry than those that are assumed; in other words, the mortality is slightly understated. The use of the nearest duration is open to a little criticism through a practical difficulty that

arises. Withdrawals occur partly from lapses, which necessarily take place when a premium becomes payable. If premiums are payable yearly the nearest duration gives an exact result if we reckon durations only to the date when the premiums fall due. In practice, however, thirty days of grace are allowed, and if an insured person dies within the days of grace the sum assured is paid. In order to make a proper comparison between the "Deaths" and the "Exposed to Risk" we should either ignore all such claims and treat every case as a withdrawal in which the premium is not paid on the day it falls due; or if we wish to include such claims among the "Deaths," we should treat the other lapses as withdrawing at the end of the days of grace, otherwise each lapse is given a month's exposure too little and the exposed to risk is underestimated. If, however, we reckon the durations up to the end of the days of grace, the following scheme shows the error that occurs in certain circumstances by assuming the nearest duration:—

Yearly Premiums . .	Each withdrawal by lapse understated by a month.
Half-yearly Premiums .	Falling due on anniversary of policy—each withdrawal understated by a month.
Half-yearly Premiums .	Falling due 6 months after anniversary—each withdrawal overstated by 5 months.
Quarterly Premiums .	Falling due on anniversary of policy—each withdrawal understated by 1 month.

Quarterly Premiums	Falling due 3 months after anniversary of policy—each withdrawal understated by 4 months.
" "	Falling due 6 months after anniversary of policy—each withdrawal overstated by 5 months.
" "	Falling due 9 months after anniversary of policy—each withdrawal overstated by 2 months.

If the proportionate distribution of cases is 12 yearly cases, 8 half-yearly, and 4 quarterly, the total understatement is 12 months in respect of yearly premium cases, 4 months in respect of half-yearly, 5 months in respect of quarterly, while the overstatements are 20 in respect of half-yearly, and 7 in respect of quarterly premiums, leaving a total overstatement of 6 months on 24 lapses. This is hardly a large error, and on the basis of Table II it would mean an error in the rate of mortality of $(\cdot 1)^5$ if every withdrawal is due to lapse, which is not the case.

The error in any individual experience depends on the proportion of yearly, half-yearly, and quarterly cases, but it is unlikely that the error involved would be large enough to make it worth while to remodel the method. A short preliminary investigation can almost always be made to settle the point.

CHAPTER III

AGGREGATE TABLES AND THEIR CONNECTION WITH SELECT TABLES

In the previous chapter we saw that the select tables we had made gave the rates of mortality for each age at which the assurances were effected and for each succeeding year during which the policies remained in force; these rates of mortality are therefore excellently suitable for the calculation of premiums. When, however, estimates of the liabilities of an assurance office have to be made, it saves labour to group together all cases of the same attained age, regardless of the time they have been in force, and for this purpose we want a table that does not show the mortality for each duration, but only the mortality for each attained age. Such tables have, however, a wider scope, as mortality tables may have to be formed from the experience of lives which have undergone no medical examination or other process of selection; for instance, the staff employed by a large manufacturing company. In these circumstances it is waste of time to calculate the rates of mortality for each year of observation in respect of each age at which the observations started; all that is required is the rate of mortality for each age during employment.

The principles involved in calculating these rates of

mortality are the same as those which have been explained. The reader will recollect that in Chapter I we said that cards were written for each case, and that these cards gave the nearest age at entry, the exact duration for those existing at the close of the observations, the nearest duration for withdrawals and the "curtate" duration for the deaths. We assumed that the nearest age and the nearest duration were approximately the exact age and the exact duration. Let us follow this assumption up and see what the result would be if it is interpreted in ages instead of durations. If a person enters at age 30 exactly and withdraws or is existing at the end of exactly 6 years, he is under observation from age 30 till age 36. Similarly, death in the sixth year means death between ages 35 and 36.

If the age at entry is 30 nearest birthday, and the duration between $5\frac{1}{2}$ and $6\frac{1}{2}$ years, we have already seen that we can treat the case as entering at exact age 30 and remaining under observation for exactly 6 years, so that in this case also the life is observed from age 30 until age 36.

Individual cases may be wrongly estimated, but in the bulk the method will be sufficiently accurate.[1]

The routine work consists of—

1. Entering on each card the age at exit, which is found by adding the duration to the age at entry.

[1] A person entering at age 30 nearest birthday and withdrawing at duration 6 (nearest), is stated to withdraw at 36. This may be a year wrong either way: *e.g.* $29\frac{1}{2}$ at entry, $5\frac{1}{2}$ duration gives an actual attained age of 35.

2. Sorting the cards according to age at entry.
3. Recording the number of cards for each age at entry (see Table IV).
4. Re-sorting the cards according to mode of exit, *i.e.* death, existing and withdrawal.
5. Sorting each of these three lots according to age at exit.
6. Recording the number of cards in the appropriate columns for each age at exit (see Table IV).

TABLE IV.

Age Attained.	Entered.	Existing at Close of Observations.	Withdrawals.	Deaths.	Exposed to Risk.	Rate of Mortality.
22	1529	...	148	5	1381	0·00362
23	1617	76	242	15	2675	0·00561
24	1532	102	298	18	3792	0·00475
25	1416	110	458	16	4622	0·00346
26	1399	133	469	41	5403	0·00759
27	1473	163	505	42	6167	0·00681
28	1518	187	549	39	6907	0·00565
29	1483	235	582	46	7534	0·00611
30	1400	266	558	60	8064	0·00744
31	1368	277	554	49	8541	0·00574
...	...	...	...	...	...	...
...	...	...	...	...	...	...

This work gives the figures in all except the last two columns of the above Table IV, which is an abstract from a larger table giving complete results for each age.

To obtain the "Exposed to Risk" we begin at the youngest age and deduct the withdrawals at that

age from the number entering. The deaths must not be deducted, because, as we have already explained, a full year's exposure is given to them: and as we are dealing with the youngest attained age, which must also be the youngest entry age, there will be no existing to trouble about. The exposed to risk at this age is thus the number entering less the number withdrawing. Deducting the deaths during the year we obtain the number continuing into the next year of age, and if we add the new entrants at this age and deduct the "withdrawals" and the "existing," we obtain the exposed to risk in the second year of age—counting from the youngest age in the experience. Continuing in this way we are enabled to fill in the figures in the last column but one of Table IV.

To make himself familiar with the method the reader should go through the table and see exactly how the exposed-to-risk column is built up from the other columns.

The rates of mortality are found by dividing the deaths at each age by the exposed to risk. The results are given in the last column and are called "aggregate" rates of mortality, to distinguish them from the "select" rates of mortality described in Chapter II.

So far we have assumed that these "aggregate" rates are found independently of the "select" rates and have no connection with them. There is, however, an intimate connection. If a complete select table is taken giving particulars, such as those in Table III, for each age at entry, the figures in the

aggregate table could be formed from those in the select table by addition. This will be made clear by an example in which we have assumed that there were only three possible ages at entry, and in order to save space have only shown the exposed to risk and deaths.

TABLE V.—SELECT TABLE

Duration.	Age at Entry, 30.		Age at Entry, 31.		Age at Entry, 32.	
	Exposed to Risk.	Deaths.	Exposed to Risk.	Deaths.	Exposed to Risk.	Deaths.
0	1469	6	1446	3	1359	9
1	1261	10	1222	10	1148	10
2	1115	7	1060	7	1010	7
3	1013	11	941	6	901	8
4	925	11	843	8	821	7
5	832	6	773	5	755	8
6	752	5	710	3	694	9
etc.	...	...	...	...	...	...

TABLE VI.—AGGREGATE TABLE, FORMED FROM TABLE V

Age.	Exposed to Risk.	Deaths.
30	1469	6
31	2707	13
32	3696	26
33	3221	28
34	2876	24
35	2576	22
36	2346	17
etc.	...	...

In order to see how the aggregate table (Table VI) is formed from the select table (Table V), we have

only to remember that a person entering at age 30 whose policy has been two years in force is then aged 32; also, that a person entering at age 31 whose policy has been one year in force, is also aged 32; and so on. Consequently, in order to find the total exposed to risk at age 32, we have to add the following exposed to risk—

Age at entry	32	duration	0
„	31	„	1
„	30	„	2

A similar method has to be adopted for the deaths.

With this explanation and the help of the two tables it is not difficult to see how "aggregate" and "select" tables are related. A special case of this relationship is of great use when we are tabulating select rates of mortality, and the various actuarial functions derived from them. In Chapter II we explained that when dealing with assured lives, it was best to work out rates of mortality in each year of assurance for each age at entry because the mortality depends on the age at which a person was examined as well as on the attained age, and Table III showed how to tabulate the rates of mortality for each age at entry and each duration. We only gave a small part of the table, as the complete table would be very large and would contain a wearisome amount of statistical information. Besides this, while it is obviously true that persons now aged 40 who were assured 5 years ago are more likely to die within a year than those aged 40 who have just been assured, one naturally asks whether there is any

difference between lives now aged 40 who were assured 5 years ago and those assured 10 or 15 years. If not, is it possible to simplify our select table?

The answer to these questions is that as the duration increases it becomes less important, and after some years it can be neglected; "selection" has worn off; this enables us to simplify our tables.

The easiest way to appreciate these points is by considering the two following tables, in which selection is shown to have worn off after 5 years, so that the rates thereafter depend only on the attained age. The first table is similar to Table III, and shows all durations, and the second is an abridged form giving the same particulars.

If we require the rates of mortality for all durations for any age at entry and have a table such as Table VIII, all that has to be done is to read off the rates across the table up to the column headed "5 or more" and then read down that column. This gives all the durations and holds good, because in the particular table selection has worn off at the end of 5 years from entry. Thus, for instance, the rate of mortality in the ninth year from entry among lives entering at age 55 is found in the last column of the table against age 63, *i.e.* ·027.

Now consider what interpretation should be placed on the column headed "five or more"; it gives the rates of mortality simply according to age for every case of more than 5 years' duration; it is, in fact, an aggregate table excluding the first five years of assurance, and we could construct it from our cards by assuming the date of entry in every case to have been

TABLE VII.—RATES OF MORTALITY (SELECT)

Age at Entry.	Duration.								
	0	1	2	3	4	5	6	7	8	Etc.
55	·007	·013	·018	·021	·023	·024	·025	·026	·027	etc.
56	·010	·016	·020	·022	·023	·025	·026	·027	...	...
57	·014	·019	·021	·022	·024	·026	·027	...	...	...
58	·016	·020	·022	·023	·025	·027	...	...	...	etc.

TABLE VIII.—RATES OF MORTALITY (SELECT AND ULTIMATE)

Age at Entry.	Duration.						
	0	1	2	3	4	5 or More.	Attained Age.
55	·007	·013	·018	·021	·023	·024	60
56	·010	·016	·020	·022	·023	·025	61
57	·014	·019	·021	·022	·024	·026	62
58	·016	·020	·022	·023	·025	·027	63

5 years later than it actually was. There is in fact no difficulty in making a table excluding any number of years of assurance; the real difficulty lies in deciding how many years to exclude.

Unfortunately we do not know when selection

wears off, as the various select tables that have been prepared show very different results in this respect; sometimes selection only lasts 3 or 4 years, sometimes as many as 10, while it seems probable that at some ages it wears off sooner than at others.

When the "select" rates are given it is possible to estimate how long selection lasts, but even then the problem is difficult as the facts are obscured by the roughness of the data, and the reader who wishes to realise its difficulties is recommended to examine the original data of some large table and attempt to estimate how long selection is appreciable in it. The problem is really one of the graduation rather than the construction of mortality tables, but a very helpful rough estimate of the duration of selection can always be made by calculating aggregate tables for the whole experience, then for the whole excluding the first 5 years of assurance, and then excluding 10 years, and seeing whether and what differences exist between the respective rates of mortality.

Table IX shows such a result.

A comparison of the columns in this table shows that selection in the particular case lasted more than 5 years; it may have lasted more than 10 years, but to prove this we should require to examine the experience excluding say 15 years. If this showed approximately the same rates as those in column (4), we should conclude that selection wore off somewhere between 5 and 10 years, and the investigation of aggregate tables excluding 9, 8, etc., years would help to show more exactly where it ended. However, as we have already remarked, this part of the subject

TABLE IX

Attained Age. (1)	Aggregate Rates of Mortality deduced from— Whole Experience. (2)	Experience, excluding First 5 Years of Assurance. (3)	Experience, excluding First 10 years of Assurance. (4)
35	·00760	·00869	·00908
36	·00778	·00883	·00934
37	·00828	·00898	·00937
38	·00846	·00921	·00894
39	·00836	·00889	·00882
40	·00916	·00979	·01026
41	·00956	·01019	·01036
42	·01014	·01077	·01074
43	·01076	·01143	·01146
44	·01134	·01191	·01241
45	·01124	·01180	·01230

is outside the scope of the present work, and it is unnecessary to pursue it further, but the reader will perhaps appreciate that it leads to interesting, though at times tedious, study.

We may conclude this chapter by summarising the uses of aggregate tables of mortality as follows:—

1. For valuation purposes.
2. For cases in which there is no selection.
3. For simplifying select tables by supplying the ultimate rates of mortality into which the select rates of mortality run.

CHAPTER IV

ALTERNATIVE METHODS

It was assumed in our work in Chapters I–III that the mortality investigation is made from the date of assurance, or, in certain aggregate tables, after the assurance has been a certain fixed number of years in force; but it is sometimes advisable to start the investigation in a certain year, and include all policies that are then in force. These policies, from the point of view of our mortality investigation, are exactly like new entrants, except that they first come under observation some time after the date of assurance. Thus if our investigation started in 1905, a policy might be included which had been taken out in 1900 by a life then aged 30, so that the policy had been five years in force in 1905; while another policy might have been taken out in 1890 at the same age at entry, so that it was 15 years in force in 1905. These cases would go into the same table as regards age at entry 30, but would be disregarded until we were dealing with the years of duration 5 and 15 respectively, when they would be brought into the Exposed to Risk for the first time.

In order to avoid dealing with fractions of a year it is usual to include in the experience each such case from the policy anniversary in the year in which the

observations commence, but if this were impossible the nearest duration would be used. The numerical work is so similar to that shown in Table II of Chapter II, that it is unnecessary to go into a detailed description of it, but the reader should see for himself how the Exposed to Risk in the following table is built up:—

TABLE X.—NEAREST AGE AT ENTRY, 30

Duration.	Entrants.	Existing at Close of Observations.	Withdrawals.	Deaths.	Exposed to Risk.
0	1499	...	30	6	1469
1	37	45	158	10	1297
2	30	27	114	7	1176
3	37	35	62	14	1109
4	40	30	53	12	1052
5	25	48	42	9	975

In Chapters II and III we assumed that our facts were so fully known that there was no difficulty in calculating the ages at entry and durations in the most convenient way, but it sometimes happens that full information is not available and our methods have to be modified.

A very simple example will give one possible modification. Nearly all Insurance Companies in this country charge premiums according to the age next birthday, and it is possible that some offices

might not be able to ascertain the exact date of birth of their assured lives without hunting up old records, but that apart from this all the other facts could be given accurately. This means that we could proceed exactly on the lines of our previous investigation, but instead of using the nearest age we should always use the age next birthday and our final figures would give the rates of mortality at age 30 next birthday, for instance, instead of at age 30 nearest birthday. In other words, since persons aged 30 next birthday are on the average about half a year less than 30, the two results would relate to ages differing by six months, and as mortality increases with the age the rates of mortality taken out for ages next birthday would be a little less than the rates taken out for ages at their nearest birthday.

Another example may be taken by supposing that the ages and the complete number of years in force are known in all cases and that the observations ended on 31st December last. The reader will see that, from an insurance point of view, the facts give the number of full years' premiums that have been paid and it might therefore in certain cases be a convenient method of showing the facts. The deaths are given as we want them, but the withdrawals and existing need adjustment, for, as the reader will remember, we ought to record them at their exact durations or nearest durations, and as they are given for the complete number of years in force their true duration is understated on the average by half a year in each case. Thus, all cases that were described as existing or as withdrawals with durations of 4 years,

for example, may really have been exposed to risk for any period between 4 and 5 years, and we must see that they are all given $4\frac{1}{2}$ years' exposure. Table XI shows how the exposed to risk would be worked out:—

TABLE XI

Age at Entry, 30.				
Number of Entrants, 1305.				
Complete Number of Years in Force.	Existing at the Close of the Observations.	Withdrawals.	Deaths.	Exposed to Risk.
0	10	85	5	1257·5
1	16	125	8	1134·5
2	16	100	6	998
3	18	53	7	898·5
4	20	30	8	831
5	30	20	9	773
6	25	16	5	718·5
etc.	etc.	etc.	etc.	...

Since each existing or withdrawal entered opposite duration 0 has really to be treated as if it had been at risk for half a year, we must only deduct half the 10 existing and half the 85 withdrawals in finding the exposed to risk at duration 0: if we deducted the whole we should assume they had not been exposed

at all. Deducting one-half of 95 from 1305, we get 1257·5 as the exposed to risk at duration 0. In order to get the exposed to risk at duration 1 we must begin by deducting the other half of the existing and withdrawals at duration 0—namely, 47·5, and we must also deduct those who died at duration 0, just as we did in Table II. Besides this, we must deduct half the withdrawals and existing at duration 1, for the same reasons as those which led us to deduct half at duration 0 when obtaining the exposed to risk at that duration. The deductions are therefore:—

1. Half the withdrawals and existing at duration 0	47·5
2. The deaths at duration 0	5
3. Half the withdrawals and existing at duration 1	70·5
Total	123
Exposed to risk at duration 0 . . .	1257·5
Exposed to risk at duration 1 . . .	1134·5

The figures for subsequent durations are worked out on the same principles, and the reader will probably have no difficulty in reconstructing the table.

Another type of approximation is necessary when, as sometimes happens, the year of birth instead of the date of birth is given, and the only other information available is the calendar year of entry, the calendar year of exit, and the cause of exit. We will assume that the observations ended on 31st December last. Let us consider the age at entry first. If the year of birth is deducted from the year of entry we

shall, on the average, get the exact age, but individual cases may be as much as a year out either way: *e.g.* if year of birth is 1860 and year of entry 1900, the extremes will be found in the cases of a man born 31st December 1860 who entered 1st January 1900, and one born 1st January 1860 who entered 31st December 1900. In the former case the true age at entry is 39, in the latter 41. The possible error in an individual case is therefore double as great as it was when using the nearest age, but on the average the result is not unsatisfactory. In just the same way the deduction of the year of entry from the year of exit gives an approximation to the true duration, and though individual cases are overstated or understated the result is not far out. The cases existing at the close of the observations are a little more difficult: any case that entered last year may have entered at any time during the year,—on the average it will have entered in the middle of the year (30th June) and it will therefore have been half a year in force when the observations ended. Cases entering the previous year and existing on 31st December last will have been one and a half years in force, and so on. To sum up, we have approximately the exact age at entry; the exact durations for deaths and withdrawals; and the exact durations for existing, but the durations for the existing will always be given as an odd half, *e.g.* $6\frac{1}{2}$ or $7\frac{1}{2}$.

The facts in this form are not very difficult to manage so far as withdrawals and existing are concerned, because the former can be used as in our work in Chapter II, and the latter as in our last example

but the deaths are not in a form that we have previously used, as they are given for their exact durations (approximately) instead of for the number of complete years in force.

In our last example we saw that the number of complete years was the same as the exact number less a half, so that in the present case we can get the number of deaths sufficiently accurately for say curtate duration 5 by taking the sum of half the recorded deaths at duration 5, and half the recorded deaths at duration 6. The first duration 0 only relates on the average to half a year, so that in finding the deaths for curtate duration 0 we must take *all* the recorded deaths for that duration and half those for duration 1. The method has automatically halved the deaths at duration 0 for us.

The following diagram may help the reader to follow the general principle—

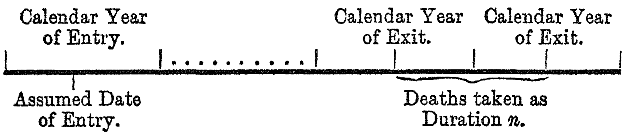


Table XII gives a numerical example.

Column (5) in this table gives the approximation for the deaths for curtate duration t by adding half the deaths for durations t and $t+1$ except as already explained for duration 0, where all the deaths at duration 0 are added to half those for duration 1.

The exposed to risk for duration 0 is found by deducting from the number of entrants (2005) the number of withdrawals (40) at duration 0 and half

TABLE XII

Duration t. (1)	Existing at Close of Observations with Duration $t+\frac{1}{2}$. (2)	Withdrawals at Duration t. (3)	Deaths at Duration t. (4)	Deaths at Curtate Duration t (from previous column). (5)	Exposed to Risk. (6)	Rate of Mortality, Col. 5÷Col. 6.
	Age at Entry, 30.					
	Number of Entrants, 2005.					
0	25	40	4	8·5	1952·5	·0044
1	32	210	9	10·5	1705·5	·0060
2	45	149	12	12·0	1507·5	·0080
3	64	73	12	11·5	1368·0	·0084
4	55	74	11	9·5	1223·0	·0078
5	54	48	8	8·0	1111·0	·0072
6	43	37	8	...	...	...

the number of existing (12·5). We only deduct half of these, because the 25 existing are to be treated as exposed for half a year each.

The exposed to risk for duration 1 is formed from that for duration 0 by deducting the deaths (8·5) for duration 0 in column (5), the withdrawals (210) for duration 1, and half the existing for duration 1 (16), and the other half of the existing for duration 0 which have not already been deducted (12·5). The total deduction is 247, and the exposed to risk at duration 1 is therefore 1705·5.

The rate of mortality is found by dividing the number of deaths for the number of complete years (curtate duration) by the exposed to risk; in this case column (5) divided by column (6).

The last method is not entirely successful if applied to the construction of Select Tables, especially in the earlier years of assurance. If there is a rapid movement in the exposed to risk the withdrawals and deaths for successive complete years in force get confused, and as the rate of mortality changes rapidly in these early years the results may be somewhat distorted. For the construction of Aggregate Tables, however, where the effect of "duration" is not considered, the method is convenient and sufficiently accurate.

Aggregate tables can be formed by the method of Chapter III from select tables constructed by the methods outlined in this chapter, and the reader can also work out how such tables could be formed directly from the original data without filling up each select table first. This exercise and the construction of examples similar to those just given will help to make the subject clearer. The method which it is best to follow is to think out carefully exactly what the particular set of facts can give and then see how they have to be adapted to enable us to use the principles indicated in Chapters II and III.

We may conclude this chapter by giving an example of the construction of an aggregate table from data in a form which is a little different from any we have yet given, but is sometimes found convenient when the investigation of the mortality of a

friendly society has to be made. We shall see in the following chapter that the method can conveniently be arranged for getting out rates of sickness concurrently with the rates of mortality, and this makes it particularly suitable in connection with friendly society work.

In previous examples each case has first come under observation on the date of assurance—true or approximate—or some anniversary of this date and has been traced through succeeding years of duration, and the data has retained this form when used for aggregate tables although age attained has been substituted for duration of assurance. When we are dispensing with select tables and wish to construct directly an aggregate table, however, there is no particular reason why we should use the age attained on a policy anniversary as the basis for our calculations. The age on any other date will do as well, provided we are consistent and observe every case during successive periods of twelve months.

We can therefore use the 1st January in some year as the starting-point of our experience, group together all lives of the same nearest age on that date, and observe them through calendar years, closing the experience on 31st December in a later year.

The entry of new members, death and withdrawal, will be assumed to take place, on the average, in the middle of a calendar year for the same reason as that which led us to assume that the withdrawals in our example on p. 30 took place in the middle of a year of duration, and as we are working on the basis of nearest ages on 1st January these movements will be

recorded as taking place at half ages, and the "existing" will be taken at exact ages.

If the experience runs from 1st January 1909 to 31st December 1913, the facts will be given in the form shown in Table XIII.

TABLE XIII

Nearest Age. (1)	Survivors on 1st Jan. 1909. (2)	Entrants during Five Years (ages calculated at previous 31st Dec.). (3)	Withdrawals during Five Years (ages calculated at 31st Dec. previous to withdrawal). (4)	Deaths (ages calculated at 31st Dec. previous to death). (5)	Existing on 31st Dec. 1913. (6)
20	25	124	12	0	0
21	95	431	45	1	106
22	156	776	69	2	190
23	222	1202	89	3	310

This table tells us that on 1st January 1909 there were 25 people aged 20 (approximately) on the books of the friendly societies with which we are concerned; during the following 5 years 124 persons entered whose ages on the previous 31st December were about 20. Some of these 124 people entered in 1909, some in 1910, and so on, and those entering in 1909 were aged 20 (approximately) on the 31st December 1908, but were only connected with the society for half the year in which they entered. Similarly, the deaths and withdrawals may relate to any one of the five years. There would not be any "existing" at the earliest age in the experience.

We can now calculate the exposed to risk of death: at age 20 it will be:—

Number of Survivors	25
Add—Half new entrants	62
	87
Deduct—Half withdrawals	6
Exposed to risk at age 20	81

To find the exposed to risk at age 21 we proceed as follows:—

Exposed to risk at age 20 . . .			81
Add—Survivors at 21		95	
Half new entrants at 20 . .		62	
Half new entrants at 21 . .		215·5	
		372·5	
Deduct—Half withdrawals at 20 .	6		
Half withdrawals at 21 .	22·5		
Deaths at 20 . . .	0		
Existing at 21 . . .	106		
		134·5	
			238
Exposed to risk at 21			319

And so on.

There is no real difference of principle between the cases we have discussed in this chapter and those previously given, and many slight variations may arise in practice, but with a little care there is no difficulty in finding suitable methods for working out the rates of mortality in any case that may occur.

In any particular experience that may have to be investigated it is well to see if the approximate methods proposed deal adequately with withdrawals where premiums are payable half-yearly or quarterly. This can be done on the lines set out at the end of Chapter II.

CHAPTER V

RATES OF WITHDRAWAL, MARRIAGE, AND SICKNESS

ALL the investigations that we have described up to the present have been made with the object of finding the rates of mortality that have been experienced, but it is sometimes necessary to find rates of withdrawal, marriage, or sickness.

Rates of withdrawal are generally required in connection with industrial insurance, friendly society business, or pension funds, and the methods are in principle the same as those already discussed, except that as we are working out withdrawal rates we must interchange "withdrawal" and "death" throughout the description of our work. In other words, the duration of the deaths will be given as an approxima-

tion to the exact duration—*e.g.* nearest duration—while withdrawals will now be shown according to the number of complete years in force (curtate duration).

In practice, when rates of withdrawal are calculated it is usually desired to get out the rates of mortality from the same experience, and if one of these methods is used, without modification, it follows that all the cases of withdrawal and death must be tabulated both according to nearest duration and curtate duration, and thus require to be dealt with twice over. As, however, rates of withdrawal are not often required very accurately, it is generally sufficient to obtain approximately the exposed to risk of withdrawal from the exposed to risk of death. The method will naturally depend on that adopted to obtain the rates of mortality, and may be illustrated by reference to the approximation given on pp. 30, 31. We saw that as a first approximation we can assume that the average duration in the final year of each withdrawal is 6 months. The same assumption can be made with at least equal accuracy for deaths, and the exposed to risk of withdrawal at duration 5, for instance, is therefore the exposed to risk of death at duration 5 decreased by half the deaths and increased by half the withdrawals at duration 5. The adjustment in other cases may become rather more complicated if it is desired to

investigate the experience according to duration, and it is often more convenient to use the same approximation in calculating both rates of death and withdrawal. For instance, adopting the assumption that deaths and withdrawals occur evenly over each year of duration and tabulating both according to "curtate" duration, we can calculate a special mean exposed to risk by including in the exposed to risk in any year one-half of both the deaths and withdrawals in that year. The addition to this figure of one-half the deaths will give us the exposed to risk of death, and the addition to the same figure of one-half the withdrawals will give us the exposed to risk of withdrawal.

Rates of marriage for each age are required in connection with the valuation of widows' funds, and for estimating the premiums to be charged for insurances against birth of issue which are wanted in some cases when reversions to property are sold or mortgaged. Rates of marriage are obtained on the same principles as those that have just been described. A special point arises, however, because, when we work out the rates of marriage, we have to decide the marriages that are to be counted, and the individuals to be included in the "exposed to risk." We might require to estimate the rate of marriage among bachelors, or the rate of marriage among single persons (bachelors or widowers), or we might want to find the rate of marriage based

on all persons, whether married or single, in the community from which our figures are obtained. In the first case we must find the ratio of the number of marriages of bachelors at each age to the number of bachelors "exposed to risk" of marriage at the same age, and by analogy with the way we have worked out rates of mortality we should use for our "exposed to risk" of marriage the number of bachelors living at the beginning of the year decreased by one-half of the deaths among bachelors, because those who died had on the average only half the year in which they might have married. If the rates of marriage for "single" persons are required, we must allow for the re-entrance of those who become widowers.

It is sometimes convenient in connection with such work as the valuation of widows' funds to use modified rates of marriage (sometimes called "central rates") based on the population at the middle of the year; by this means the same "exposed to risk" can be used in finding rates of marriage and rates of mortality, but in any subsequent work on the rates obtained in this way, care must be taken to interpret the figures appropriately, and not to assume that they are identical with the ordinary rates of mortality, etc., as defined in our first chapter.

There is no distinction in principle between finding

rates of mortality and rates of withdrawal, but when we come to deal with rates of sickness we have to remember some points in which such investigations differ from mortality and similar investigations.

In the first place the "rate of sickness" is not strictly analogous to the rates of mortality and withdrawal. It is *not* the ratio of the number of persons falling sick within one year from the attainment of a specified age to the number who were under observation for one year from the attainment of that age. This ratio, which we may call the "rate of incapacitation," is often very useful, but it is not what is commonly known as the "rate of sickness." The latter function depends on the number of weeks sickness, and has sometimes been taken as the ratio of the number of weeks of sickness to the number alive at the beginning of the year. The exposed to risk by this method is the same as the exposed to risk of death. The accepted definition at the present time of the rate of sickness at any age is the average number of weeks sickness experienced by those under observation during one whole year following the attainment of that age, and is found by dividing the total number of weeks sickness at any age by the average number alive during the year. This last number may be calculated in the same manner as the special mean exposed to risk of p. 41, or by tabulating all the deaths and withdrawals according to their nearest duration, as an approximation to the exact duration, the effect of either method being to observe cases of death and withdrawal up to the date of death or withdrawal, and not up to the end of the year.

The rate of sickness is very important in friendly society work, as the allowance in sickness is a payment, usually weekly, during incapacity, and not merely a sum down for each illness. It is for this reason that we require to work on the basis of the amount of sickness, and not on the number of people falling sick. We must also remember that a person may receive sickness benefit several times, and does not necessarily pass out of observation after once receiving benefit, as he does of course in the case of a mortality experience.

In consequence of these differences, investigations have to be made on slightly modified lines, and it is necessary to record how many weeks' payments have been made in each case. A further complication arises in practice, because friendly societies pay a scale of benefits varying with the duration of the illness. These scales vary considerably. Under the National Insurance Act, 1911, for instance, there is no benefit for the first three days of sickness; then sickness benefit is allowed for 26 weeks, and subsequently a "disablement benefit" of a reduced amount is paid —"disablement" benefit being another name for reduced sickness pay.

Some friendly societies grant benefits which decrease at the end of 3 months, again at the end of 6 months, and perhaps again at the end of 12 months, but it will suffice if we deal with the rates of sickness in two groups only, namely, "first 26 weeks' sickness" and "subsequent sickness"—the principle being the same if there are more subdivisions.

As a consequence of the practice of reducing the sickness benefit in this way it is necessary to calculate the rate of sickness for each period of attack, and in our final table we shall have a series of columns showing for each age the rate of first period sickness, *i.e.* the average number of weeks per person during which the full sickness benefit was paid; second period sickness—*i.e.* the average number of weeks during which the first reduced benefit was paid; and so on.

In making investigations into sickness rates a different form of card is used from that adopted for mortality investigations, and the specimen opposite will be found to give the information required.

If we are dealing with the experience of a single society granting the same benefits to all its members, the columns for the dates on which sickness began and ended may be dispensed with, and the figures in the columns for first 6 months and subsequent sickness will be the number of weeks' full and reduced pay respectively, shown against the members' names in the society's claim register. For a larger or more varied experience, however, the former columns are necessary, and the sickness in each period will be calculated from an inspection of the dates there recorded. In this connection it is necessary to notice a very usual rule among friendly societies, that any illnesses not separated by a fixed minimum period (for Approved Societies under the National Insurance Act it is one year) are treated as though they were continuous for the purpose of deciding what benefit is to be paid. In the case of any illness, therefore, the sickness benefit

Name **Member's Number**

Occupation

Date of Birth

„ Entry **Age at Entry**

„ Freedom **„ Exit**

„ Exit **Mode of Exit**

Year.	Age.	Illness Began.	Illness Ended.	Sickness.			
				First Six Months.		Thereafter.	
				Weeks.	Days.	Weeks.	Days.

NOTE.—*In nearly all friendly societies no sickness benefit is allowed until after the expiration of a certain period, and the date of freedom means the date when such allowance may begin.*

paid may have been that for either the first or second period or for both, and the number of weeks' sickness in respect of that illness may run over several ages.

As an example, let us suppose a man born 8th August 1884 to have had the following sickness experience :—

Fell Ill.	Recovered.
12th January 1909.	13th April 1909.
10th November 1909.	10th March 1910.
6th February 1911.	6th March 1911.
4th April 1912.	13th January 1913.

If the rules of his society provide for 6 months' full benefit and reduced benefits thereafter, with an "off" period, as it is called, of 52 weeks, the actual benefit paid by the society would be as follows :—

TABLE XIV

During	Full Pay.		Reduced Pay.	
	Weeks.	Days.	Weeks.	Days.
First illness . .	13	...	...	...
Second ,, . .	13	...	4	1
Third ,, . .	...	...	4	...
Fourth ,, . .	26	...	14	3[1]
Total . . .	52	...	22	4

[1] Benefit is commonly paid in respect of working days only. This should be remembered in reading the following Table XV.

The manner in which this is entered on the experience card will depend on the method adopted in investigating the experience and calculating the exposed to risk.

Let us assume that the experience of the five years 1909–1913 is under investigation and that nearest ages are dealt with—the member coming into the experience on 1st January 1909 at his then nearest age. Bearing in mind that this is an approximation to the exact age, so that we must assume age 25 to be attained on 31st December 1909, age 26 on 31st December 1910, and so on, we see that the following entries (Table XV) must be made on the card:—

TABLE XV

Year.	Age.	Sickness.			
		First Six Months.		Thereafter.	
		Weeks.	Days.	Weeks.	Days.
1909 . . .	24	20	2	...	...
1910 . . .	25	5	4	4	1
1911 . . .	26	...	...	4	...
1912 . . .	27	26	...	12	4
1913 . . .	28	...	...	1	5
Total	. . .	52	...	22	4

4

These entries can be very easily understood. The assumption of the nearest age at 1st January 1909 leads to the further assumption that calendar years and years of age are concurrent, so that we simply require to calculate the amount of sickness of each period occurring in each calendar year and enter the result on the card against the appropriate year of age.

Thus we have assumed that the man in question was 24 last birthday throughout 1909, 25 last birthday throughout 1910, and so on. In 1909 he had one illness of 13 weeks, and a second illness, which lasted during the last 7 weeks and 2 days of that year and continued for 9 weeks and 5 days in 1910. The amount of "first period sickness" at age 24 was thus 13 weeks plus 7 weeks 2 days, or 20 weeks 2 days, and, as less than one year separated these attacks, 5 weeks and 4 days only (the balance of the 26 weeks sickness payment allowed) of the continued illness at age 25 in 1910 would be paid for at the full rate, the balance of 4 weeks and 1 day being paid for at the reduced rate. The third illness occurred in 1911, within 12 months of the second, and reduced pay only was received, therefore, throughout its duration of 4 weeks, which must be entered against age 26 on the card. The last illness commenced more than 12 months after the expiration of the previous sickness and full benefit was again due for 26 weeks; it continued for 38 weeks and 4 days in 1912 and for 1 week 5 days at the beginning of 1913. We must therefore record 26 weeks at full pay and 12 weeks 4 days at reduced pay against

age 27, and 1 week 5 days of reduced pay against age 28.

One special point requires mention in connection with the apportionment of the sickness among the different periods of attack. As the experience of five calendar years only is being investigated, we shall find that the payments made during the first illness recorded on some of the cards will be affected by previous illnesses, and a special note must be made of such cases. This can generally best be effected by placing a mark against the date of commencement of such illness, when the card is written, with a note at the bottom of the card pointing out the exact circumstances. Attention would be drawn to such cases when the cards are written by the fact that reduced sickness benefit would have been paid either from the commencement of such illness or before it had continued for 26 weeks.

When dealing with mortality investigations we explained that the number of deaths, withdrawals, etc. at each age or duration could be found by sorting the cards and counting them, but when judging the total amount of sickness at each age to be compared with the exposed to risk, a little more trouble has to be taken, because one card may give sickness for various ages. If the experience extends only over a few years the difficulty can be met, by filling up all other particulars and then separating the cards on which any sickness is shown. These can then be sorted according to the first age at which sickness appears, and the total amount of sickness found by addition either by sight or, if the numbers are large,

by an adding machine and then carrying forward each card to the next higher age where sickness is shown. In many cases where the experience is large, auxiliary sheets will have to be used for tabulating the sickness.

As an illustration of the calculation of the rate of sickness we may take the last example in Chapter IV, the further particulars required being shown in Table XVI. Columns 2, 3, 4 and 6 of Table XIII are omitted to save space, but it will be noticed that we could calculate rates of mortality and sickness in the same table.

TABLE XVI

Nearest Age. (1)	Exposed to Risk of Death. (2)	Deaths. (3)	Exposed to Risk of Sickness. (4)	Total First Period Sickness in Weeks (ages calculated at previous 31st Dec.). (5)	Total Second Period Sickness in Weeks (ages calculated at previous 31st Dec.). (6)	Rate of First Period Sickness. (7)	Rate of Second Period Sickness. (8)
20	81	0	81	40	3	·49	·04
21	319	1	318·5	240	24	·75	·08
22	830·5	2	829·5	650	80	·78	·10
23	1650·5	3	1649·0	1280	170	·78	·10

As already explained, the "deaths" are given their exact duration in calculating the exposed to risk of sickness, and this will include on the average half a year's exposure in the year of death. Each such case

is counted for a full year in the exposed to risk of death, and we must therefore deduct one-half of the figure in column (3) from the figure in column (2) at each age. The result is given in column (4). The figures in columns (5) and (6) are obtained, as shown above, by adding up the total sickness at each age recorded on the cards, and the rate of sickness for the different periods is found by dividing these figures by the figures in column (4).

The practice of the different friendly societies varies considerably, and a number of conditions modifying the payment of a sickness benefit are met, each of which necessitates some modification of the methods outlined in this chapter. For instance, sickness benefit is frequently not allowed for the first six months of membership, but a death benefit may be payable although the member die within the six months. In this case each new entrant would enter into the exposed to risk of death and withdrawal at entry, but would not enter into the exposed to risk of sickness until six months later, and the method of working has to be modified. Other rules of a more troublesome nature may also occur, but if the fundamental principles of calculating the exposed to risk are borne in mind, no difficulty should arise in any individual case.

CHAPTER VI

MORTALITY TABLES FROM CENSUSES AND DEATH REGISTERS

In the preceding chapters we have dealt with the treatment of data from which we can obtain the history of each individual case, and the approximations resorted to for the sake of convenience may be relied upon to give very accurate results.

We may now consider briefly the problem of obtaining the rate of mortality from census records and mortality returns where it is impracticable to observe individual cases. We must still keep clearly before our minds that our final object is to obtain the rates of mortality at each age, and as we cannot obtain exactly the elements in our formula we must endeavour to find a suitable basis for approximation. For this purpose we may go back to our definition in Chapter I and consider the numerical example in which we imagined 10,000 persons aged 30, each of whom was kept in sight until he attained age 31 or until death, if it occurred before 31. We concluded that if there were 104 deaths the rate of mortality at age 30 was ·0104. If these deaths occurred regularly during the year we could dispense with the enumeration of the people at the outset, as by counting those surviving to age 31 or to any age

between 30 and 31, and adding the deaths after age 30 up to that time we should obtain the number commencing at age 30. Let us assume that the figures related to the calendar year 1912, and see what would have been the position of the community of 10,000 persons on 1st July when 6 months of the year under consideration had elapsed. Fifty-two persons would have died, and there would remain a population of 9948 persons aged $30\frac{1}{2}$ years. A census taken on that date would give this figure, and we could deduce the exposed to risk by adding 6 months' deaths, *i.e.* $9948 + 52 = 10{,}000$, and the rate of mortality would be $104 \div 10{,}000 = \cdot 0104$ as before. This shows us that if we are given the number of persons aged $30\frac{1}{2}$ alive on 1st July in any year, and the number of deaths during the same year at age 30 last birthday, we can find the rate of mortality.

In practice, of course, it is quite impossible to trace a number of people, all born on 1st January in the same year, but if we take the population aged 30 last birthday on 1st July and the deaths aged 30 last birthday during the year, we can say that *on the average* the birthday will be 1st January both for population and deaths, and our results will be very close to the truth. Similarly, we cannot find a stationary population, but we may assume that the population in the middle of the year will be the average population living during the year; in other words, we shall arrive at approximately the same figure if we take the result of a census on 1st July as if we traced the members of the population

individually and made allowance for the time they were members of the community the mortality of which we are investigating. By taking the population on 1st July instead of 1st January we should thus in ordinary circumstances overcome the difficulty of finding a number of persons born on the same day, and we should also make allowance for variations in the population during the year due to migration and increase or decrease consequent on a changing birth-rate in former years.

Put generally, then, the rate of mortality at any age is equal to the number of deaths in one year divided by the population at the middle of the year increased by half these deaths, the people included in each count being those who attained the age in question on their last birthday.

The method we have outlined, although making some allowance for variations, assumes that there has been no violent fluctuation in the number of people in the community, or that variations before and after the middle of the year when the census is taken counterbalance. In such circumstances the method will give good results, but it is awkward to apply in practice, because censuses are not made yearly on 1st July, and it is preferable to average the deaths over a number of years to avoid fluctuations due to epidemics, etc. These difficulties can, however, be overcome to a very great extent if the population is not subject to erratic variations by using one census and the average number of deaths for three years. This simple method often gives satisfactory results, but an alternative method is to take the

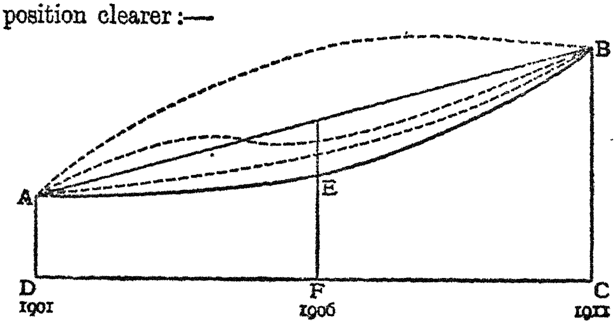
mean population based on two censuses for the population in the formula for the rate of mortality, and the deaths used are those occurring during the period between the two censuses. The reader will see at once that by using more than one census it is easier to decide whether the population is changing, and to what extent. An extreme case will show the danger of using only one census: Consider, for instance, a newly populated district in some out-of-the-way part of the world which was unpopulated up to July in a certain year, when immigration started, and 100 people settled in the district, and before the end of the year two of them died. If a single census had been taken at the middle of the year, it would appear that there was no population, and yet two deaths would have been recorded! This is, of course, a ridiculously extreme case, but the use of two censuses and the deaths for the intervening period would avoid a bad error even here.

Our next problem, then, is to find the mean population during the period between the dates of the two censuses, and we must first see exactly what we understand by this term. It is simply the average of all the populations there have been during the period. If we want to find the batting average of a cricketer we add together all the runs he has made and divide the result by the number of completed innings. In the same way we should get our mean population by adding together all the populations and dividing by the number of enumerations. Unfortunately, however, while a cricket innings is complete in

itself, and is divided by well-marked intervals from other innings, a population is in a state of constant change and varies from day to day on account of deaths and removals. In order to obtain an exact result, therefore, we should have to take a census every moment, whereas in actual practice the census —at least in this country—is taken only once in ten years! We are thus faced with the difficulty of deciding from the results of two censuses, separated by an interval of ten years, what has been the population at different times within the ten years, or, which is practically the same thing, in what manner the population has varied.

It will be seen at once how awkward this is: because it would be very difficult to guess a mean population correctly from these two censuses alone. Some idea might perhaps be obtained from the deaths, because the deaths depend on the population; but in practice the helpfulness of the deaths is discounted to a large extent by their fluctuations, which disguise variations due merely to the changes in the population.

The following diagram may help to make the position clearer:—



A D and B C represent the population of a community in 1901 and 1911 respectively, and we want to find the average population during the period. Another way of setting the problem is to say that we know A D and B C and we want to know the area of the whole block A B C D, but do not know how the line A B ought to be drawn. We could draw it in so many ways. We could make it a straight line; or a curve, running either above or below the straight line; or we could make it twist about in all sorts of ways. A few of the many possible solutions are shown in the diagram by dotted lines. Think how different the area is in almost every case, and how unlikely we should be to guess the right distribution of population which is shown by the complete line. The problem would be very much easier if we were given E F—the population in 1906—but unfortunately this is not available.

The result of this difficulty has been that people have "theorised," and the theory was set out that the population increased in geometrical progression. This theory seems to rest on the idea that population begets population, and that therefore if the population is 100 this year and double this number next year, it will again double itself the next year, and so on. The reader must remember that this is simply assumption, but it is convenient in so far as it enables us to find the population at any moment, and so obtain the sum of all the populations without much trouble. In some cases where it can be tested it seems not unreasonable. It can be very far out in other cases, and a decrease in marriage-rate, postponement of age of

marriage, a changing death-rate, or many other causes, could upset the geometrical progression. This can be shown by taking a somewhat extreme case. There are certain populations which are now decreasing, although at one time they were increasing. At some time between say 1903 and 1913 the population changed from an increasing one to a decreasing one. The population was in fact like the top line of our figure, and the geometrical progression which is below the straight line gives a worse result than the straight line which represents an arithmetical progression.

A geometrical rate of increase is, however, frequently assumed, and the consequent mean population is $P \cdot \dfrac{r-1}{\log r} \times (\cdot 434 \ldots)$, where P is the population at the first census, rP the population at the second census, and the logarithm is the common logarithm to base 10.[1]

If, however, instead of assuming a geometrical progression it is assumed that the population was increasing in arithmetical progression, the mean is one-half the sum of the populations at the two censuses, and in cases in which the assumption is correct this is the same as the population at the middle of the term.

In order to test the effect of these different

[1] The formula is obtained by the application of the integral calculus. On the given assumptions the population at any time is Pr^t, and the mean population during the ten years is therefore $\int_0^1 Pr^t dt = P \cdot \dfrac{r-1}{\log_e r}$.

assumptions let us consider the case of a population of 10,000 on 1st January 1901 which had increased to 20,000 by 1st January 1911. The mean population would be 14,428 on the assumption of a geometrical progression and 15,000 on the assumption of an arithmetical progression, and if we further assume that 2250 deaths occurred during the ten years 1901–1910 the corresponding rates of mortality would be ·0155 and ·0149 respectively, showing a difference of 4 per cent. In calculating these rates we must take one-tenth of the 2250 deaths, because these deaths relate to ten years. This is an extreme example, of course, but it shows that the assumptions made may have considerable influence on our results.

So far we have assumed that we are dealing only with one age, but in practice we generally have to use the numbers living between certain ages and not the number at each age, and we have therefore to calculate mean populations for these groups as well as for the total population, and we have afterwards to spread out the data so that rates of mortality can be found for every age. It will seem at first sight that there is little difficulty in the first point, because we can calculate the mean population for each group independently; add up these mean populations and treat the total as the mean for the population as a whole.

If we assume that the population in each age group is varying in arithmetical progression, then the sum of the separate mean populations obtained for the different groups will be equal to the mean

population found from the total number living at the two census dates.

If, however, we work on the basis of a population increasing in geometrical progression, we shall find that unless the rate of increase in every group is the same, we shall not necessarily obtain this equality. This result is only to be expected when the nature of the assumption made is considered, and does not, in itself, condemn the method of taking the geometrical mean population of each age group separately. It has, however, been considered by some a defect in the method, and in the Population Tables (1911) in this country the assumption of a geometrical progression has been made with regard to the total population only, the mean population for the different age groups being obtained by assuming that the proportion of the population in any group to the total varies in arithmetical progression. It can be seen that by this device the objection is overcome; and further, that if we work on the ratios in this manner we shall always get equality between the sum of the mean grouped populations and the mean of the whole population, no matter what assumption we make as to the manner in which the population as a whole is varying.

Having thus calculated the mean population for the groups of ages given in the census returns, we have now to spread out the results in some way or other so as to obtain deaths and populations for each age on which to base our rates of mortality.

Table XVII gives the facts for a small portion of a certain experience.

TABLE XVII

Ages.	Population.	Deaths.
30–40	7472	79·1
40–50	7312	104·9
50–60	5012	91·5
60–70	3728	153·8
70–80	1628	135·1
80–90	496	87·1
90–100	84	24·9
100–105	16	3·6

The data are shown graphically on a small scale in Figs. A and B. The rectangular blocks are propor-

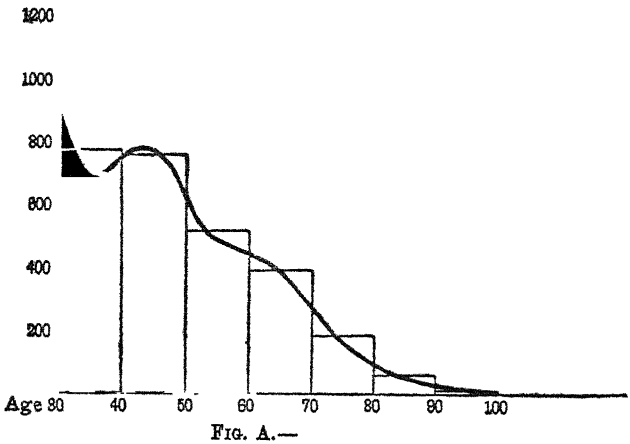


FIG. A.—

tional to the populations and deaths, and the lines running through them enable us to distribute the

figures over the ages in the groups. As we are merely distributing we must see that the line drawn does not lessen the size of any block—in other words, the population read off from figure A for any one of the original groups will be the same whether we use the curved line or the straight one, but the population

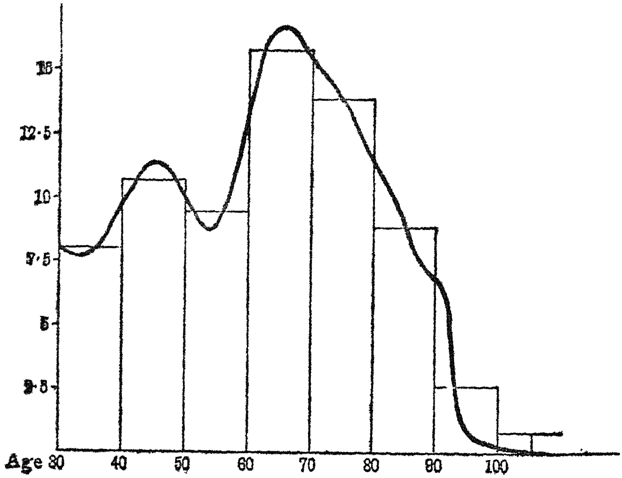


Fig. B.—Deaths.

for any smaller age group will differ. The objections to the method are (1) that it is quite possible to draw many lines satisfying the necessary condition, and each line gives a different distribution and different values for the rates of mortality; (2) that it is quite likely that the figures obtained will not run very smoothly, and may require some adjustment afterwards if the results are to be useful in practical calculations; (3) that it is not easy to read off the areas

from the curve with sufficient accuracy unless the drawing is on a very large scale. The method is, however, so simple that it has often been used; sometimes with considerable success. The Carlisle table was calculated on these lines by Joshua Milne in 1815, and the method is sometimes called by his name.

Another method is to assume that the rate of mortality deduced from the group, is the rate for the central age of the group and to obtain other values by interpolation. This method is short and easy, but it is objectionable because it introduces systematic errors. These errors arise because the mortality increases with the age in such a way that the rate of mortality for the group does not usually give the rate for the central age of the group.

The method generally adopted is to interpolate by means of one or other of various algebraical methods that are available between the figures given for populations and deaths, and so produce figures that can be used for individual ages.

Alternatives are available in the application of the formula chosen. We may either split up the population in each age group to find the number at each age, or we can find the total population over certain ages by summation of the grouped figures, interpolate to find the population over each age, and then, by differencing the results thus obtained, arrive immediately at the number living at each age. Thus, in the example given on page 63 we can find the population at age 52 by dividing 5012 into 10 parts in some manner, or by summing the figures in column

2 to obtain the total population over 50, over 60, over 70, etc., interpolating to find the total population over 51, over 52, over 53, etc., and taking the difference between the total population over 52 and the total population over 53. The latter method is more convenient and, with the modification described below, is usually employed. It might also be adopted with the graphical method referred to above, but its utility is discounted by the immense scale on which the work has to be done to enable the calculator to read off from the graph the additional figures involved. On the other hand, it would simplify the reading from the graph, because we should read off the heights of the curve at each age instead of calculating the areas.

We may save a little more work yet in arranging the function for interpolation. The reader will remember that the rate of mortality or chance of dying in a year is—

$$\frac{\text{Deaths}}{\text{Population} + \frac{1}{2}\text{ deaths}}$$

the chance of living a year is therefore the difference between this and unity, namely—

$$\frac{\text{Population} - \frac{1}{2}\text{ deaths}}{\text{Population} + \frac{1}{2}\text{ deaths}}$$

One advantage of this second form over the first is that the numerator and denominator are more alike, and the same method of interpolation can therefore more properly be used for the two functions; another advantage is that a certain amount of

arithmetic is saved because we only calculate "Population $+ \frac{1}{2}$ deaths" and "Population $- \frac{1}{2}$ deaths" for a few age groups, and the interpolation and differencing then give the numerator and denominator for finding the probability of living a year without any further calculations. If we work with the deaths and populations we have afterwards to calculate the "Population $+ \frac{1}{2}$ deaths" for each age, which entails over 100 calculations.

Even the process of interpolation is not altogether easy, as we have to complete a long series of values, and it would be impracticable to use all the given terms for this work. Interpolations are therefore made from various selections of terms, and the final results are artificially blended, or, preferably, the ordinary methods of interpolation are discarded and a method is used which, though a little artificial, gives continuity without the trouble of any independent blending process.

The above methods give good results for the main portion of a mortality table, but they are unsuitable for the first few years of life. During infancy and childhood the rate of mortality changes very rapidly, and it is found that the ages of children are frequently stated inaccurately, so that it is practically impossible to adopt satisfactorily any method of spreading the facts recorded, for the first groups of ages. The exposed to risk have consequently to be calculated from the recorded births and deaths. We will assume that the deaths at each age are available for the ten years 1901 to 1910, and that they can be obtained for years prior to 1901 if required. Of the

deaths at age 0, *i.e.* those occurring before age 1, among children born in the year 1900, some will have occurred in 1900 and the remainder in 1901; and of the deaths at the same age among children born in 1901, some will have occurred in that year and the rest in 1902. Approximately, then, the deaths occurring in 1901 at age 0 must have been one-half of the total deaths under age 1 among children born in 1900 and 1901, or we may say that the exposed to risk corresponding to the deaths at age 0 in 1901 will be one-half of the births in the two years 1900 and 1901.

Similarly with other years: so that we conclude that the deaths under one year of age for the ten years 1901 to 1910 arose out of an exposed to risk made up of half the births in 1900, all the births in 1901 to 1909 inclusive, and half the births in 1910.

Now let us turn to age 1. Among the children born in any year, some will die before reaching age 1, and we must therefore deduct these in finding the exposed to risk for that age. Otherwise the method for finding the exposed to risk at age 1 is the same as that for age 0, except that the births must be taken for one year earlier. Consequently we reach the following as the exposed to risk at age 1, corresponding to the deaths at age 1 during 1901 to 1910: one-half of the births in 1899, all the births in 1900 to 1908 inclusive, and one-half of the births in 1909, less the deaths under age 1 in 1900 to 1909.

When we come to the next age, deductions must be made for the deaths of two ages, and so on. The result may be set out as follows:—

Exposed to risk corresponding to deaths during 1901 *to* 1910

Age.

0. $\frac{1}{2}$ births in 1900 + births in 1901 to 1909 + $\frac{1}{2}$ births in 1910.
1. $\frac{1}{2}$ births in 1899 + births in 1900 to 1908 + $\frac{1}{2}$ births in 1909 − deaths at age 0 in 1900 to 1909.
2. $\frac{1}{2}$ births in 1898 + births in 1899 to 1907 + $\frac{1}{2}$ births in 1908 − deaths at age 0 in 1899 to 1908 − deaths at age 1 in 1900 to 1909.
3. $\frac{1}{2}$ births in 1897 + births in 1898 to 1906 + $\frac{1}{2}$ births in 1907 − deaths at age 0 in 1898 to 1907 − deaths at age 1 in 1899 to 1908 — deaths at age 2 in 1900 to 1909.
4. $\frac{1}{2}$ births in 1896 + births in 1897 to 1905 + $\frac{1}{2}$ births in 1906 − deaths at age 0 in 1897 to 1906 − deaths at age 1 in 1898 to 1907 — deaths at age 2 in 1899 to 1908 − deaths at age 3 in 1900 to 1909.

The exposed to risk found in this way never agrees exactly with the population given by the census. This is partly because the exposed to risk gives the number at an exact age, while the population gives the number for all ages—in other words, the exposed is, as we have already seen, equal to the population with half the deaths added. We must therefore in the first place compare the total exposed to risk at these five ages, less half the deaths, with ten times the mean

population found from the census figures. They will be unlikely to agree owing to the population not being stationary, emigration, immigration, etc. Since the deaths and population must correspond, however, the figures found above have to be modified to bring them into line, and this is done by increasing or decreasing each exposed to risk in the same proportion, so that their total after adjustment will be the same as the mean population under age 5 in the census returns increased by half the deaths. The deaths during the census period at each age divided by the exposed to risk so modified will give the rates of mortality.

This completes the calculation of the rates of mortality from census data, and we may conclude the chapter by summarising the method adopted :—

1. Set out the population from two censuses and the deaths for the period between the two censuses in the age groups available.
2. Calculate the mean population for the total population.
3. Calculate the mean population for each age group, so that the mean population for the total populations found in (2) is reproduced.
4. Spread out the mean population and the deaths so as to give particulars for each age.
5. Calculate the rate of mortality at each age by assuming that it is equal to the deaths divided by mean population plus half the deaths.

6. For the early ages, instead of using (4) and (5), calculate the exposed to risk by tracing the children born, and adjust the results to make them agree with the mean population. The rate of mortality is the deaths divided by adjusted exposed to risk.

An alternative method is sometimes used with satisfactory results when the population is not subject to violent fluctuations, and consists of working on a single census, and the deaths for, say, three years (see p. 56).

CHAPTER VII

APPLICATION OF CENSUS METHOD TO INSURANCE DATA

Before discussing the actual application of the census method to insurance data it will be advisable to point out more definitely than has yet been done the essential difference between the census method and the insurance methods previously described. In the latter the deaths recorded at any age are those occurring among the individuals included in the exposed to risk at that age. In the census method we start with the number of deaths registered at each age (or age group) and find a corresponding population, but although these figures give an accurate approximation to the rate of mortality, the "population" is calculated from the results of two censuses which are unrelated. If, as a basis for calculating the mean population, we make one census of all the people aged 30 last birthday on 1st January 1912, and another of all the people aged 30 last birthday on 1st January 1913, no person would appear in both these groups, because the people who were aged 30 on 1st January 1912 will be aged 31 on 1st January 1913. Disregarding migrations, the deaths, used in estimating

the rate of mortality at age 30, occur out of the two groups which are gradually changing throughout the year; each day during 1912 some of the people who were aged 30 last birthday on 1st January pass their 31st birthday, while some of the people who were aged 29 at the beginning of 1912 reach their 30th birthday.

The nearest census equivalent to the insurance method would perhaps be to count the people aged 30 last birthday on 1st January 1912, and those aged 31 on 1st January 1913, and the deaths during 1912 between ages $30\frac{1}{2}$ and $31\frac{1}{2}$. This is more awkward with census data, and would be less accurate, especially when censuses are taken at intervals of 5 or 10 years. The method adopted in census work for the very early ages is similar to the insurance method, but in the present chapter the expression "census method" will be employed to refer to that adopted for the main portion only of a mortality table formed from population statistics.

Let us now recapitulate the "census method" and see what it has to commend it. If we have two censuses giving the population for each age last birthday on 1st January 1912 and the 1st January 1913, and if we know the number of deaths at each age last birthday during 1912, the rate of mortality at any age is found by dividing the deaths at that age by the mean population increased by half the deaths. In practice, then, if we take two censuses and the deaths, we can calculate, by a small amount of arithmetic, the rates of mortality for each age if the results of the censuses, etc., are available for each age

and have not been grouped. The method entails far less work than that given in the earlier chapters, where each individual has to be followed for a number of years, and although it is apparently less accurate the result obtained is a sufficiently close approximation. But the chief recommendation of the census method is that it enables us to find in a comparatively short time the rate of mortality that has been experienced, while the other method is so lengthy that the results, when reached, are somewhat out of date. This is so objectionable in practice that it is well worth while to consider the possibility of applying the census method to life office investigations.

In the simplest form this is not very difficult. The insurance office could make a "census" of all the people who were assured on, say, 1st January 1920, and again on 1st January 1921. It would have from its records all the deaths for the intervening period, so that the rates of mortality it has experienced could be found very quickly for each age. Those rates would be "aggregate" rates of mortality, but the method could easily be extended to give "select" rates.

If the office set out at each age—(1) the number of assured people who were in the first year of assurance on 1st January 1920, (2) the number who were in the first year of assurance on 1st January 1921, and (3) the number of deaths during 1920 that occurred in the first year of assurance, then the rate of mortality for the first year of assurance could be obtained in the same way as the aggregate rate. Similar results could be reached for any other year of

assurance, or for all the policies which have been more than, say, 5 years in force.

A numerical example will show the working; we have only given in it two years in the select period so as to save space, but in practice this term would have to be increased to five or possibly ten years.

TABLE XVIIA

Age Last Birthday.	Number of Policies on Books (*i.e.* Population) having on 1st Jan. 1920 a Curtate Duration of			Number of Policies on Books (*i.e.* Population) having on 1st Jan. 1921 a Curtate Duration of			Number of Policies going off by Death in 1920 having at Death a Curtate Duration of		
	0	1	2 or more	0	1	2 or more	0	1	2 or more
40	122	98	603	133	105	625	6	8	60
41	131	92	550	120	111	560	7	7	61
42	110	92	575	122	91	580	4	6	55
43	115	90	560	118	93	560	5	5	58
44	101	90	525	106	84	535	5	6	56

Age.	Mean "Population" plus one-half of "Deaths."			Rates of Mortality for Exact Age.		
	0	1	2 or more	0	1	2 or more
40	130·5	105·5	644	·0046	·0076	·0093
41	129	105	585·5	·0054	·0067	·0104
42	118	94·5	605	·0034	·0064	·0091
43	119	94	589	·0042	·0053	·0099
44	106	90	558	·0047	·0067	·0100

The top part of this table gives the information obtained from the books of the office or offices concerned. It tells us that on 1st January 1920 there were 823 policies (122 + 98 + 603) on the books on the lives of people aged 40 last birthday, and of these 122 had been issued in 1919 and were less than one year in force on 1st January 1920, 98 had been issued in 1918 and were more than one but less than two years in force, and the remaining 603 were issued before 1918 and were more than two years in force. A year later there were 863 policies (133 + 105 + 625) on the lives of persons aged 40 last birthday, subdivided into 133 of less than one year in force, 105 of more than one and less than two years in force, and 625 of more than two years in force. The last three columns of this part of the table give the deaths. They tell us, for instance, that in 1920, 74 policies (6 + 8 + 60) became claims, owing to the deaths of lives assured who died when they were aged 40 last birthday, and there is the appropriate subdivision according to the duration of the policies at the time of death.

The way the information is utilised is shown in the lower part of the table: the mean "population" for age 40, duration 0, is one-half of (122 + 133) or 127·5, and the addition of one-half of the six deaths gives 130·5. The rate of mortality is $6 \div 130 \cdot 5 = \cdot 0046$. Similarly, the other figures are found.

In practical work the experience of a single year, even if many insurance offices combined, would be too small to give a table suitable for general use, although a series of such tables might be useful in showing the effect on mortality of yearly fluctuations, epidemics,

etc. We should therefore amalgamate a series of tables similar to that shown in Table XVIIA if we wished to obtain rates of mortality suitable for the general purposes of an insurance office.

When describing the method, we have hitherto assumed that the returns would be made for ages last birthday, and we made this assumption because it shows the method in a form which is, as nearly as possible, alike to the actual form of the facts disclosed by censuses and death registers. There is, however, no need to restrict ourselves in this way, and as offices do not usually group their policies according to the ages last birthday of the lives assured, some variation will probably be found necessary. There are many ways in which the facts may be available, and each one can be used if the deaths are taken properly. If, for example, an office groups policies according to the nearest ages of the lives assured on 31st December, then the returns would most conveniently give the numbers on the books at 31st December at the nearest ages which, as we have already seen, is an approximation to the exact ages. If the deaths during the year are also stated at their nearest ages at death, the method explained in Table XVIIA will give rates of mortality for each half-year of age—or for age last birthday if the reader prefers to think of it in that way. The subdivision into the years in force is not in any way disturbed.

In order to give the deaths according to the age nearest birthday at death, an office that groups its policies according to the nearest age at the end of each year would have to rearrange the deaths in different groups from those in which they appear in

the books. This inconvenience may be avoided. If the deaths are stated according to the nearest age at the beginning of the year in which death occurred, they are automatically recorded at their approximate ages last birthday at date of death. The two "populations," and therefore the mean "population," are stated at the exact (nearest) ages. If we take the mean of the resulting figures for consecutive ages, say 30 and 31, we reach a mean population at 30 last birthday in the middle of the year, and can use the ordinary census method. An office can easily give the curtate durations for the populations, as they depend on the year of entry. The deaths must be given according to the curtate durations at death.

Other methods can be devised, and, if the points already indicated are borne in mind, no great difficulty should arise.

It may seem at first sight that the application of the census method to life assurance data is a little rough and ready as compared with the methods explained in our earlier chapters, but a little consideration shows that it should give approximately accurate rates of mortality. So far as ages are concerned the fundamental assumptions are the same in all methods, and as the deaths are recorded at their correct ages and durations it is hard to see how error can creep in, unless for some reason the use of the arithmetical mean of the "populations" at the beginning and end of the year plus one-half of the deaths becomes incorrect as the divisor for finding the rate of mortality. This might happen if the "population" varied in an extreme way or if the rate of mortality at any age is

changing rapidly. The changes in "population" can be detected easily, and if there were extreme variations we could base our "mean" population on three enumerations instead of two. The other source of inaccuracy indicated may appear at any age or duration if the mortality is changing rapidly at that age or duration. This can only happen at the older ages or in the first year of assurance, and even then can have little influence if the population is approximately constant.

When, however, we compare the census method as applied to life assurance data with the detailed census method described in Chapter VI, we find that we gain considerably, as we have accurate particulars for each age and therefore save all the grouping and redistribution of the facts into age groups, which makes the census method complicated, and as we can make our censuses whenever we like from the office's valuation books, we can avoid the difficulties connected with the calculation of the mean population. In practice a single office can arrange its books so that the "censuses" required can be obtained each year automatically, at any rate for whole life assurances; but for other classes,[1] or for a large combined experience, it might be inconvenient to make these censuses at frequent intervals, but by making one initial census, and adjusting it to allow for subsequent alterations, a continuous census can be arranged. Such continuous methods could be evolved for the investigation methods of Chapters II, III, and IV, but they lend

[1] A practical method for endowment assurances and special classes which will work well with some offices is to keep the cards which are employed in connection with valuations in order of the year of birth, although the policies are grouped in another way, such as the year of maturity. Enumeration is then simple, and with a continuous valuation no difficulty arises.

themselves peculiarly well to the census plan, and as they are of practical value it is advisable to show how the facts can be conveniently arranged to give a continuous mortality investigation with the minimum of labour.

The advantages of an efficient continuous system are that results are obtained quickly, and to ensure this it is essential that the information required should be easily given, avoid detail, and lead to results that are sufficiently accurate for all necessary purposes.

In any continuous system we require the data at some chosen starting-point and particulars of the various alterations to which these data have been subjected. This means, in the present case, that we want first a record of the number of policies to be investigated in the following form (Table XVIII).

TABLE XVIII.—NUMBER OF POLICIES IN FORCE ON 1ST JANUARY 1912

	DURATION.										
Age last birthday on 1st Jan. 1912.	Less than 1 year in force.	More than 1 and less than 2.	2–3.	3–4.	4–5.	5–6.	6–7.	7–8.	8–9.	9–10.	Over 10 years in force.
..	..	..		..	..	..	..	..	..	..	..
..	..	..	.	..	..	..	..	..	.	..	..
..	..	..	..	..	..	..	..	..	..	..	..
..	..	..	..	..	..	..	..	..	..	..	..
54	500	396	373	368	354	356	339	336	327	320	12,250
55	512	401	385	377	368	364	353	340	325	318	10,260
56	501	485	400	390	384	369	365	338	336	325	9,127
..	..	..	..	..	..	..	..	..	..	..	..
..	..	..	..	..	..	..	..	..	..	..	..
..	..	..	..	..	..	..	..	..	..	..	..
..	..	..	..	..	..	..	..	.	..	..	..

During the period of investigation each new policy effected and each old policy discontinued from any cause must be noted, and the figures in Table XVIII adjusted so that we arrive finally at the popu-

lation at the end of the period without the necessity for another "census" enumeration.

The adjustment can best be carried out by writing the Date of Entry, Date of Birth, Date of Exit and Cause of Exit, and Policy Number on a card, similar to that already illustrated, whenever a movement takes place, and by sorting the accumulated cards periodically and then making the necessary adjustments.

The manner in which these cards are to be sorted calls for special remark. As they are to be used to find the population at the *end* of the period in each year of age and duration, we must give consideration in every case to the age and policy year *at that time*, and not at the time the movement takes place.

For instance, a policy is effected on 1st March 1912 by a man who will be 50 on 1st June 1912. He is therefore 49 last birthday at entry, but on 1st January 1913 he will be 50 last birthday if he survives, and we must count him as of that age for the purpose of getting the population aged 50 last birthday in the first year of assurance on 1st January 1913.

Similarly, a policy effected on 1st October 1901 by a man born 1st June 1862, is lapsed by the non-payment of the premium due 1st April 1912. At this date the man was 49 last birthday and the policy was between 10 and 11 years in force, but we must deduct this case from those between 11 and 12 years in force and on lives aged 50 last birthday on 1st January 1913 in estimating the population at that date.

The whole operation may be explained by saying

that to get the population at 1st January 1913 aged 50 last birthday in the fifth year of assurance we put down the population on 1st January 1912 aged 49 last birthday in the fourth year of assurance, and deduct those of these particular cases who go out of observation during the year.

The population at 1st January 1913 aged 50 last birthday in the first year of assurance will, of course, relate to policies effected during 1912 on lives that will be 50 last birthday on 1st January 1913, less any discontinuances during the year out of these new policies.

Tables XIX and XX, illustrating the method, should be carefully studied and the results calculated independently. The particulars for two consecutive ages have been inserted so as to indicate the continuity of the process. It will be noticed that policies over ten years in force have been grouped together for convenience and to save time and space. As already pointed out, it is impracticable and unnecessary in most cases to trace the effects of selection for more than ten years, but if this is needed the Tables merely require to be extended and the particulars further analysed.

From the results given in these tables we can find the mean population, but we must not use the number of claims recorded in them to find the exposed to risk. The deaths recorded in Table XIX, for example, are those occurring among people who were all 55 last birthday on 1st January in some year: the deaths with which we are concerned in calculating the exposed to risk are those arising among people

TABLE XIX.—SHOWING POPULATIONS EACH YEAR. AGE LAST BIRTHDAY, 55

	Duration.										
	0-1.	1-2.	2-3.	3-4.	4-5.	5-6.	6-7.	7-8.	8-9.	9-10.	Over 10.
Population * 1st January 1912 (see Table XVIII), age 55 last birthday .	512	401	385	..	..	..	..	..	..	318	10,260
Carried forward from previous duration, aged 54 last birthday on 1st January 1912	506†	500	396	..	..	..	..	..	..	330	12,250 ‡ 320 — 12,570
Less—											
Withdrawals in 1912 § out of carried forward	20	60	50	..	..	..	..	..	..	18	57
Claims in 1912 § out of carried forward	6 — 26	15 — 75	10 — 60	..	..	..	..	..	..	12 — 30	180 — 237
Population 1st January 1913, age 55 last birthday	480	425	336	..	..	..	..	..	..	300	12,333
Carried forward from previous duration, age 54 last birthday on 1st January 1913	515†	470	440	..	..	..	..	..	..	350	11,195 360 — 11,555
Less—											
Withdrawals in 1913 out of carried forward	18	52	40	..	..	..	..	..	..	15	48
Claims in 1913 out of carried forward .	7 — 25	8 — 60	8 — 48	..	..	..	..	..	..	11 — 26	170 — 218
Population 1st January 1914, age 55 last birthday	490	410	392	..	..	..	..	..	..	324	11,337
Etc.	..	etc.	..	..	..	..	..	..	..	etc.	etc.

* For sake of clearness we have used the word "Population"; alternatively, "existing" might be used.

† Number of policies effected.

‡ The first entry, 12,250, represents the carried forward from 54 durations over 10; and the second entry, 320, the carried forward from 54, duration 9-10.

§ For the purpose of finding the population aged 55 last birthday at the end of the year from the population aged 54 at the previous duration, we must deduct only those who died or withdrew among the number in the population carried forward. This is easy to follow if it is borne in mind that we are merely using the figures as book-keeping entries to enable us to find the population at the end of the year. The claims, etc., so deducted may be among some people aged 54 last birthday, and some 55 last birthday at death, and will be different from the claims appearing in the fraction giving the rate of mortality; these latter are all the claims during the calendar year for the duration and age last birthday under consideration (see Table XXI).

TABLE XX.—SHOWING POPULATIONS EACH YEAR. AGE LAST BIRTHDAY, 56

	Duration.										
	0–1.	1–2.	2–3.	3–4.	4–5.	5–6.	6–7.	7–8.	8–9.	9–10.	Over 10.
Population 1st January 1912, age 56 last birthday	501	485	400	..	..	..	..	..	..	325	9,127
Carried forward from previous duration, age 55 last birthday on 1st January 1912	495	512	401	385	..	..	..	..	..	..	10,260 318 10,578
Less—											
Withdrawals in 1912 out of carried forward	15	65	55	..	..	..	..	..	..	..	61
Claims in 1912 out of carried forward .	4	28	7	..	..	..	..	..	..	..	150
	19	93	62								211
Population 1st January 1913, age 56 last birthday	476	419	339	..	..	..	..	..	..	..	10,367
Carried forward from previous duration, age 55 last birthday, on 1st January 1913	482	480	425	336	..	..	..	..	..	..	12,333 300 12,633
Less—	..	..									
Withdrawals in 1913 out of carried forward	..	..	..	..	..	..	..	..	..	..	55
Claims in 1913 out of carried forward .	etc.	etc.	..	..	..	..	..	..	..	..	185
											240
Population 1st January 1914, age 56 last birthday	..	..	..	..	..	..	..	..	..	..	12,393
Etc.	..	etc.	..	..	..	..	..	..	..	..	etc.

aged 55 last birthday at the date of death, who may have been either 54 or 55 last birthday on the preceding 1st January. A separate record must therefore be kept of claims arising by death as shown in Table XXI, the numbers being entered on the basis of age and duration at the time of death.

TABLE XXI.—CLAIMS ARISING AT AGE 55 LAST BIRTHDAY AT DEATH

Calendar Year of Death.	Duration at Death.										
	0–1.	1–2.	2–3.	3–4.	4–5.	5–6.	6–7.	7–8.	8–9.	9–10.	Over 10.
1912 .	5	5	7	..	..	..	..	..	..	6	220
1913 .	3	6	6	..	..	..	..	..	..	8	242
1914 .	4	4	9	..	..	..	..	..	..	4	239
1915 .	4	7	8	..	..	..	..	..	..	6	221
Total .	16	22	30	..	..	..	..	..	..	24	922

We can now find the rate of mortality experienced in any year by taking the mean of the population shown in Table XIX or XX on the 1st January of that year and the 1st January in the succeeding year; adding one-half the deaths in the year found from Table XXI, and dividing the deaths by the result. Thus the rate of mortality during 1913 at age 55 and in the third year of insurance was, on the basis of these tables, $\frac{6}{\frac{1}{2}(336+392+6)}$, *i.e.* ·01635. Or we can find the rate of mortality that has been effective during a certain number of years by adding

together the mean populations during each of these years and comparing the result with the deaths during the same years.

It is convenient to make the calculations in the manner shown in Table XXII.

The method here described was devised to facilitate the work of conducting a continuous investigation into the mortality experience of one or more life offices, and is capable of considerable adaptation to varying circumstances. Thus, it can be conveniently and usefully employed for an ordinary life office investigation into the experience of the past, where cards similar to that illustrated in Chapter I have been written for each case under observation during a fixed period.

In this case, if the calculation of rates of mortality on this plan is the sole object for which the cards are written, it is not necessary when we are working in calendar years to insert dates of birth, entry, and exit on every card; the year only is sufficient, except for death cases, where dates are still required. The age last birthday on 1st January in any year is the difference between the year of birth and the year just ended, and similarly the year of duration current on 1st January in any year is the difference between the year of entry and the year just beginning. Thus a policy effected in 1896 on the life of a man born in 1862 will be in its 16th year of existence on 1st January 1912, at which date the man's age last birthday must be 49, and whether it is maintained in force or allowed to lapse during 1912, we require no further information as to age or duration. If it

TABLE XXII.—POPULATIONS ABSTRACTED FOR FINDING THE RATE OF MORTALITY.
AGE 55 LAST BIRTHDAY

DATE.	DURATION.										
	0–1.	1–2.	2–3.	3–4.	4–5.	5–6.	6–7.	7–8.	8–9.	9–10.	Over 10.
1st January 1912	512	401	385	..	..	..	..	..	..	318	10,260
,, 1913	480	425	336	..	..	..	..	..	..	300	12,333
,, 1914	490	410	392	..	..	..	..	..	..	324	11,337
,, 1915	500	430	375	..	..	..	..	..	..	312	9,465
,, 1916	508	433	409	..	..	..	..	..	..	306	11,228
Total	2490	2099	1897	..	..	..	..	..	..	1560	54,623
Less ½ first term	256	200·5	192·5	..	..	..	..	..	..	159	5130
Less ½ last term	254	216·5	204·5	..	..	..	..	..	..	153	5614
	510	417	397	..	..	..	..	..	..	312	10,744
Mean Population	1980	1682	1500	..	..	..	..	..	..	1248	43,879
Total Deaths, 1912–1915	16	22	30	..	..	..	..	..	..	24	922
Exposed to Risk	1988	1693	1515	..	..	..	..	..	..	1260	44,340
Rate of Mortality	0·00805	0·01299	0·01980	..	..	..	..	..	..	0·01905	0·02079

becomes a claim, we must then know the dates of birth, entry, and exit, in order to calculate the age last birthday and the duration current at the moment of death.

The routine work is as follows:—

1. Turn out the cards for all entrants after the date of commencement of the observations.
2. Sort the remaining cards according to age last birthday and year of assurance current at that date.
3. Tabulate the results and so obtain P_1.
4. Replace the cards for new entrants and turn out those for all cancelments during the period.
5. Sort the remaining cards according to age last birthday and year of assurance current at the end of the period.
6. Tabulate the results and so obtain P_2.
7. Turn out from the total number of cards all cases of death.
8. Sort the death cards according to age last birthday and year of assurance current at death and so obtain *d*. Then the rate of mortality at any age and in any year of duration will be

$$\frac{d}{\frac{1}{2}(P_1+P_2+d)}$$

By repeated sortings we can obtain the population at different intervals during the period and proceed as in Table XXII.

A little consideration will show that by adopting this method we reduce our work to a minimum and obtain the results for exact ages as well as exact durations: we secure as great accuracy as we wish, and can investigate the experience of a limited and stated period.

The reader in trying to follow the method will be struck by the detail rather than the simplicity: this is almost inevitable when an attempt is made to describe verbally the details of statistical work, but if the underlying principle that we are building up a succession of censuses is borne in mind, many of the apparent difficulties will disappear and be replaced by the feeling that the method is not awkward in practice. It must also be borne in mind that the special method described in the last ten pages is only to be used when it is impossible to obtain directly from the books of an insurance office the populations and deaths at each age and in each year.

CHAPTER VIII

COMPARISON OF MORTALITY TABLES AND OTHER MISCELLANEOUS NOTES

If it is necessary to compare the mortality experienced by two insurance offices, or by two towns, or by an insurance office with that of the general population, the obvious procedure is to calculate the rates of mortality for each age and see which set of rates is higher. This is the natural course to adopt, but in many cases it is not an altogether satisfactory method. The number of cases at various ages being small, there are large accidental fluctuations in the rates of mortality, and the comparison of two such series of fractional numbers is not usually easy. The first difficulty could be got over by graduating the rates of mortality (that is by adjusting the rates and smoothing out the unevenness), but this entails a great deal of work and lays the result open to the criticism that the differences or similarities are due, to some extent, to the graduation.

The comparisons most frequently required are between the rates of mortality experienced by an insurance office, or some other relatively small population (*e.g.* members of a trade or small district), and

the rates of mortality prevailing among insurance offices generally or in the population of the whole country. Both these latter rates are known; they have been worked out and tabulated, and form the bases for all kinds of investigations. In examining the experience of an insurance office, the method adopted is to calculate the exposed to risk from that experience and to multiply the figures so found by the rates of mortality in the standard table with which the comparison is to be made. The result gives what is called the "expected number of deaths," and this is compared with the actual number of deaths. This method saves a considerable amount of work, as it avoids the calculation of the rates of mortality, and it can frequently be shortened by grouping the exposed to risk at 5 or 10 ages together, and multiplying by the rate of mortality for the central age of the group instead of working on each age separately.

Table XXIII gives an example of a comparison between the experience of the first five years of assurance of the business of an office and the mortality shown by the $O^{[M]}$ Table—which gives the experience of British offices under whole life with profit assurance in the form of select mortality tables.

Several ages at entry have been grouped together, but every duration has been given, the reasons being that more work is saved in finding the exposed to risk, and that as the differences in the rates of mortality in the early years of assurance are greater for successive durations than at consecutive ages, better results are generally obtained by grouping the ages at entry rather than the durations.

TABLE XXIII

Duration.	Ages at Entry																
	18–32.					33–47.					48–62.					18–62.	
	Exposed to Risk. (1)	Actual No. of Deaths. (2)	Expected No. of Deaths. (3)	Rate of Mortality in experience. (4)	Rate of Mortality O [M]. (5)	Exposed to Risk. (1)	Actual No. of Deaths. (2)	Expected No. of Deaths. (3)	Rate of Mortality in experience. (4)	Rate of Mortality O [M]. (5)	Exposed to Risk. (1)	Actual No. of Deaths. (2)	Expected No. of Deaths. (3)	Rate of Mortality in experience. (4)	Rate of Mortality O [M]. (5)	Total Actual No. of Deaths at all Ages.	Total Expected No. of Deaths at all Ages.
0	1710	8	4·8	·00468	·00281	2015	12	8·8	·00595	·00438	1100	14	11·4	·01272	·01040	34	25·0
1	1150	7	5·3	·00608	·00463	1500	9	9·9	·00600	·00660	791	12	11·2	·01517	·01416	28	26·4
2	840	6	4·5	·00714	·00532	1200	10	9·3	·00833	·00773	605	12	10·2	·01983	·01692	28	24·0
3	560	5	3·2	·00892	·00576	925	9	8·0	·00972	·00862	450	8	8·9	·01777	·01955	22	20·1
4	450	4	2·7	·00888	·00614	800	7	7·6	·00875	·00948	375	11	8·3	·02933	·02226	22	18·6
Total	. . .	30	20·5	..	..	..	47	43·6	..	..	..	57	50·0	.	..	134	114·1

The rates of mortality are for the central ages at entry (*e.g.* age 25 in the first group), and the expected number of deaths was found by multiplying the exposed to risk by the rate of mortality in column 5.

An examination of the table shows that in nearly every group the actual number of deaths exceeds the number expected, and that the greatest differences occur in the earliest age group, and, if all ages are taken together, at duration 0.

The rates of mortality in column 4 have been calculated for the groups as they stand. It will be seen that a comparison of these rates with those in column 5 is not so easy to make as a comparison of the figures in columns 2 and 3, nor does it give so good an idea of the difference between the mortality table as a whole and the standard table.

As a second example we may compare the mortality experienced by consumptive patients (males) who were admitted to a certain sanatorium with the mortality shown by the English Life Table No. 6, a table giving recent rates of mortality for the general population. Tables XXIV and XXV show the comparisons: the former table relates to early (incipient) cases, and the latter to advanced cases. The expected deaths were obtained on the basis of the rates of mortality from the English Life Table, but the rates actually used have been left out to save space. The tables are given in the form of select tables, although the comparison is being made with a table in which there is no selection, because it seemed probable that the rate of mortality would be heavier in the early years and lighter afterwards—just the opposite

TABLE XXIV.—MORTALITY OF MALE CONSUMPTIVES—INCIPIENT CASES

Age at Admission . .	Under 23.			23–27.			28–32			33 and over.			All Ages.		
Total Number admitted .	67.			77.			44.			34.			222.		
Number of Years since Admission.	Exposed to Risk.	Actual No. of Deaths.	Expected No. of Deaths.	Exposed to Risk.	Actual No. of Deaths.	Expected No. of Deaths.	Exposed to Risk.	Actual No. of Deaths.	Expected No. of Deaths.	Exposed to Risk.	Actual No. of Deaths.	Expected No. of Deaths.	Exposed to Risk.	Actual No. of Deaths.	Expected No. of Deaths.
0	32	..	0·14	38	..	0·21	22	..	0·14	15	..	0·14	107	..	0·63
1	64	1	0·30	76	2	0·44	42	3	0·29	34	1	0·37	216	7	1·40
2	46	2	0·28	64	1	0·38	35	1	0·25	24	1	0·28	169	5	1·14
3	38	1	0·20	50	2	0·31	28	..	0·22	18	..	0·24	134	3	0·97
4	26	1	0·14	36	..	0·23	21	..	0·17	16	2	0·22	99	3	0·76
5	12	..	0·07	22	..	0·14	11	1	0·10	11	..	0·17	56	1	0·48
6	5	..	0·03	8	..	0·05	3	..	0·03	4	..	0·07	20	..	0·18
7	1	..	0·01	1	..	0·01	..	..	..	1	..	0·01	3	..	0·03
Total	224	5	1·12	295	5	1·77	162	5	1·20	123	4	1·50	804	19	5·59

TABLE XXV.—Mortality of Male Consumptives—Advanced Cases

Age at Admission . .	Under 23.			23–27.			28–32.			33 and over.			All Ages.		
Total Number admitted .	65.			77.			67.			88.			297		
Number of Years since Admission.	Exposed to Risk.	Actual No. of Deaths.	Expected No. of Deaths.	Exposed to Risk.	Actual No. of Deaths.	Expected No. of Deaths.	Exposed to Risk.	Actual No. of Deaths.	Expected No. of Deaths.	Exposed to Risk.	Actual No. of Deaths.	Expected No. of Deaths.	Exposed to Risk.	Actual No. of Deaths.	Expected No. of Deaths.
0	37	..	0·16	41	..	0·23	34	..	0·22	45	..	0·54	157	..	1·15
1	62	7	0·29	69	2	0·40	65	2	0·44	82	7	1·06	278	18	2·19
2	48	2	0·24	57	3	0·34	43	2	0·31	61	3	0·85	209	11	1·74
3	32	2	0·17	34	3	0·21	32	1	0·25	46	6	0·69	144	12	1·32
4	18	2	0·10	22	3	0·14	25	4	0·21	30	4	0·50	95	13	0·95
5	11	..	0·06	15	..	0·10	11	1	0·10	17	3	0·28	54	4	0·54
6	7	..	0·04	4	..	0·03	6	..	0·06	8	.	0·13	25	..	0·26
7	2	..	0·01	2	..	0·01	..	..	..	..	..	..	4	..	0·02
8	..	..	..	1	..	0·01	..	..	..	..	..	..	1	..	0·01
Total	217	13	1·07	245	11	1·47	216	11	1·59	239	23	4·05	967	58	8·18

to the select mortality in life assurance data. Duration 0 relates to approximately half a year only.

It is obvious from these tables that the consumptives suffered from a very heavy mortality, and that the actual number of deaths is far greater than the "expected"; but the tables are given not to bring out this point, but to show how much preferable in the case of a small experience is the comparison of actual with expected deaths, than a comparison of the actual rates of mortality with those of the standard table. The rates of mortality, calculated by dividing the actual deaths by the exposed to risk, would have been very large at some ages and durations and nothing at all at others, so that the effect of calculating them would probably only have confused, and, as they cannot properly be added together, no general idea of the relative mortality would have been obtained.

With the figures actually given we can, however, form a very fair idea of the excessive mortality, and, taking the tables as a whole, we can say that the actual number of deaths was 3·4 times the expected number in the incipient cases and 7 times in the advanced cases. We could also say that the mortality among the incipient cases is 4·5 times as great as the expected when lives come to the sanatorium at an early age—under 23—but the ratio decreases when the age at admission is older. A similar decrease is noticeable in the advanced cases. The mortality is also more excessive in the early years after admission than in the later years.

These tables teach us something else. They show

that the method can be used to compare two mortality experiences without actually calculating the rates of mortality for either. Thus we could in the present instance compare the incipient and advanced cases, seeing that the former showed a mortality which in the total was 3·4 times and the latter 7 times that of the English Life Table. The advanced cases in the total, therefore, had a mortality about twice as heavy as the incipient. Care is of course needed in stating and interpreting such results, as, since the excess of mortality depends on the age at admission and the duration, two sets of data might give different results merely owing to the different distribution of the cases. Such points must always be borne in mind: the comparison must be criticised carefully, and apparently obvious conclusions must not be given as the final word until other possible influences have been excluded.

It is well to accentuate this difficulty, and perhaps the easiest way to do so is to remind the reader of the select tables and the aggregate tables made from them. A comparison of the mortality of an insurance office with that shown by an aggregate table might give the impression that the office was experiencing an exceptionally light rate of mortality, although the mortality was really heavier than the select rates given by the data from which the aggregate table was formed.

The aggregate rates of mortality at any age being a combination of the rates of mortality for all durations, will be heavier than the light select rates shown at early durations, and lighter than the heavier select

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rates shown at late durations. In the British Offices ($O^{[M]}$) experience, the rates were as follows :—

TABLE XXVI

Age Attained.	Rates of Mortality.			
	Aggregate.	First Year of Assurance.	Second Year of Assurance.	After Ten Years.
20	·00399	·00261	·00434	·00664
30	·00584	·00312	·00493	·00757
40	·00900	·00438	·00637	·00986
50	·01504	·00746	·00991	·01546

If an assurance office had been newly formed and had no policy on its books more than two years in force, Table XXVII might show the condition of the office.

A comparison of the actual deaths with those expected by the aggregate table would lead one to say that the actual number of deaths was only about three-quarters of the number expected, and the impression would be given that the office was being fortunate in its mortality experience, whereas in reality the mortality was about one and a quarter times as heavy as the expected, if the recent selection of the cases is allowed for.

For some purposes, however, it is correct to ignore selection and use an aggregate table in judging the

effect of mortality on the finances of an insurance office, but in reviewing the results so obtained the purposes for which the investigation is made must be kept in mind, and it must not be assumed that a light mortality revealed in this way proves that the

TABLE XXVII

Attained Age.	Number of Years in Force.	Number of Policies.	Actual Number of Deaths.	Expected Number of Deaths by Select Rates of Mortality.	Expected Number of Deaths by Aggregate Rates of Mortality.
20	0	1,000	3	2·6	4·0
...	1	500	2	2·2	2·0
30	0	3,000	12	9·4	17·5
...	1	2,000	11	9·9	11·7
40	0	2,000	12	8·8	18·0
..	1	1,000	10	6·4	9·0
50	0	1,000	10	7·5	15·0
...	1	500	6	5·0	7·5
Total	. . .	11,000	66	51·8	84·7

particular office is selecting its cases with special care or is obtaining proposals from a long-lived part of the community. The most common of these purposes is the analysis of the surplus shown after a valuation of liabilities has been made by an aggregate table. In such a case all the work must be consistent, and if

the valuation is made by an aggregate table and the analysis of the surplus brought out is made on some other basis, *e.g.* on a select mortality basis, we shall either fail to trace where the surplus comes from, or trace more surplus than that shown by the valuation. To put the matter in another way, an office might for certain reasons assume purely arbitrary rates of mortality for valuation purposes, and, if it did so, it would necessarily have to analyse its apparent surplus on the same arbitrary assumptions.

We may now turn to a somewhat different point. A mortality table may be constructed from insurance office data by tracing each life assured or each policy, or each £100 assured, or each £10 of annual premium paid, or on some other basis, and for certain purposes any one of these bases might be of use. Probably in most cases it is simplest and accurate enough to use policies and to work out rates of mortality on this basis: so that if a man is assured under 20 policies he will be counted 20 times. This may at first sound strange, but in the bulk it makes little difference, and, so far as select tables go, the method is as satisfactory as counting him only once, because his policies would not necessarily all have been taken out at the same time, and he was therefore a "select" life at several ages. Some of the others who assured on the first occasion with him may have died or become ill, but he did not. If we wished to trace each £100 assured, a man taking out a policy for £100 would be counted as one case, and a man taking out a policy for £2000 would be counted as 20 cases in calculating the exposed to risk and the rate of mortality.

While, however, each of these various systems may be useful, nearly all the well-known tables of mortality have been constructed by working out the exposed to risk on the basis of lives or policies; and when once these rates are published and the monetary tables calculated it is not necessary to trouble about the underlying method of construction when using the tables. At the same time, when a particular office wishes to study the mortality it has experienced, it is customary to find the number of policies expected to become claims by death and the amount of sum assured expected to fall due for payment by a standard table, just as we found the expected deaths when discussing the mortality of consumptives. Such work, though of value and of general interest, does not tell the actuary the profit or loss from mortality, for the insurance office has against each policy a certain amount of reserve in hand, and when a policy becomes a claim the payment of the sum assured is met out of this amount and the balance out of income. The amount of reserve depends on the age at entry, and duration of the policy, and arises owing to the office receiving a level premium for an increasing risk. An office expects to pay something out of each year's premium income towards claims, and the profit or loss from mortality depends on whether the amount actually paid out of income is less or greater than the amount expected. Let us consider the case of 100 policies in a certain office assuring £100,000 in all on lives aged 50, on which the reserves on say the $O^{[M]}$ Table with 3 per cent. interest would amount to £30,000 at the end of the year if all the lives

survived. Then as the rate of mortality for age 50 by the same table as that used in the valuation is ·015, the contribution out of income that the office would expect to have to pay is (100,000 − 30,000) × ·015 = £1050. If there were no claims, £1050 would be the profit. If there were two claims for policies assuring £1500 with reserves of £600, the actual contribution from current income is £900 and there is a profit of £150. The term "death strain" is generally used instead of "contribution from current income," and in practical work its investigation is somewhat more complicated than is implied by our example. The investigations of a life office have to be made at the end of its financial year, and the reserves have to be taken at that date both for the calculation of the actual and expected death strain, and allowance must be made in the expected strain for policies coming on and going off the books during the year. The most obvious approximation is to start with the policies in force at the end of the year, or rather the difference between the sums assured and reserves connected with them, add half the corresponding values for the surrenders and lapses and the whole of the corresponding values for the claims, and deduct half the values for the new business. This gives a good approximation, and the final groups are multiplied by the rates of mortality for the age on the previous 31st December. The reason for adding the whole of the difference between the sum assured and reserve for the claims, is the same as that for giving a full year's exposure in the year of death (see Chapter I): if only half a year's exposure is given, the expected

strain would have to be worked out by some function other than the rate of mortality. An imaginary example for policies on lives aged 51 at 31st December 1913 may assist:—

Total sum assured in force at 31st December 1913 . . .		£100,000
Reserve in respect thereof . .		30,000
Difference . . .		£70,000
Lapses—		
Sum assured . .	£2100	
Reserve thereon .	60	
Difference .	£2040	
Add one-half of £2040 .		1,020
Surrenders—		
Sum assured . .	£5000	
Reserve thereon .	300	
Difference . .	£4700	
Add one-half of £4700 .		2,350
Claims—		
Sum assured . .	£2000	
Reserve thereon .	1000	
Difference . .	£1000	
Add the whole of £1000 .		1,000
		£74,370

Brought from previous page	.	£74,370
New Business—		
Sum assured .	. £5000	
Reserve thereon	. 100	
Difference .	. £4900	
Deduct one-half of £4900	.	2450
		£71,920

Rate of mortality at age 50 = 0·015
Expected death strain = 71,920 × 0·015 = 1079
Actual death strain (claims above) = 1000

Profit from mortality in group = 79

Even if the figures are set out in less detail and in more convenient form, a method of this kind for each age group entails a large amount of work, and in practice rougher approximations are used, such as taking the mean of the groups at the beginning and end of the year and multiplying by a modification of the rate of mortality, viz., the central death-rate (ratio of deaths to population).

The reader must remember that the profit or loss from mortality recorded by the process we have described is a valuation profit or loss, depending on the method of valuation adopted; it does not for the reasons given on p. 101 prove that the particular office has experienced a lighter or heavier mortality than other offices, or that it has had a lighter mortality than that assumed in the calculation of its premiums,

unless the valuation is made by the same table as that on which the premiums were calculated, which very rarely happens.

We may revert to our example for a moment to point out that working out the expected amount of claims in sum assured instead of the expected death strain might lead us to think there had been a loss from mortality, when the reverse was the case. In our example the "exposed to risk" in sums assured is:—

In force on 31st December 1913 .	£100,000
Add Half lapses . . .	1,050
Half surrenders . .	2,500
Whole claims . . .	2,000
	£105,550
Deduct Half new business .	2,500
Exposed to risk . .	£103,050

Multiplying this by ·015 we have £1546 as the expected amount of claims, while the actual amount (£2000) is greater. The explanation is that in this particular case the claims fell on cases with reserves above the average reserves of the group.

Before leaving the subject of this chapter another method of comparing mortality which is frequently adopted in connection with Census Statistics, and is in principle analogous to the calculation of the expected deaths, may be mentioned. This method consists of assuming a standard population and calculating the number of deaths that would occur if it were subject

to the same rates of mortality as are experienced in the populations to be compared. The standard population adopted could be the same as the general population. The method would be of use for comparing the mortality in various districts of the country or the mortality in different trades. The difficulty it is intended to overcome is the different age distributions of the populations, but if the final total of deaths only is taken into account the result may not be conclusive, because it may be made up of groups some of which show excess and some defect, indicating that at certain ages the mortality of one population is the greater and at other ages is the less. This is also of importance when the expected number of deaths is used for comparison. The following example illustrates the point:

TABLE XXVIIA

Age.	Exposed to Risk.	Actual Deaths.	Actual Rates of Mortality. ×100.	Rates of Mortality of Standard Table. ×100.	Expected Deaths by Standard Table.
30	7,500	35	·47	·6	45
40	10,000	90	·9	1·0	100
50	15,000	330	2·2	2·0	300
60	15,000	555	3·7	3·5	525
70	10,000	575	5·75	6·0	600
80	2,500	235	9·4	10·0	250
Total	60,000	1,820			1,820

A comparison of the total actual and expected deaths does not reveal the fact that the actual mortality is much lighter than the standard at first, then heavier, and finally lighter again. An examination should always be made of comparatively small age-groups as well as of the totals.

CHAPTER IX

RATES OF MORTALITY AND SICKNESS BY VARIOUS TABLES

It is interesting to compare the rates of mortality that have been observed at different times among the general population and among that special class of the general population that seeks the protection of insurance, and as such study is also useful in impressing on the mind the rates of mortality at various ages and the influence of selection, we shall devote this concluding chapter to a consideration of the results that have actually been obtained.

We shall confine our attention to the published tables, which are, or have been, of general use and shall not discuss the many tables constructed from time to time by various methods from miscellaneous data. Much of the work done in this direction was valuable at the time, and some of the tables have assumed a more or less historical interest, owing to the adoption in them for the first time of an improvement in the methods of construction; but it is impracticable to go into such detail here, and anyone who is interested in the subject on the historical side can obtain all the information he wants with comparatively little difficulty.

To bring together, in full, even a few of the most important published tables would require a prohibitive amount of space and would make the comparison very laborious. In this chapter, therefore, the rates of mortality are given at a few selected ages only, and—in the case of insurance office tables—at two or three durations of insurance. The tables examined were based on a large amount of data and, generally, give the rates of mortality, etc., after graduation, *i.e.* after adjustment to remove accidental irregularities in their general progression—and they can be compared as they stand without using the method of "expected deaths." The ages have been so chosen as to indicate the general nature of each table, and the trend of the rates of mortality at the intervening ages may be judged from the examples given. The rates of sickness and withdrawal that have been found to prevail in a few large experiences are also included.

It must be pointed out that the cause of the changes shown is not always apparent, and care must be taken not to draw wrong conclusions. For example, the change that has taken place in the mortality among insured persons depends not only upon the mortality of the community as a whole, but also on the selection of lives by the life insurance offices and on the policy of these offices, which may have altered sufficiently to have attracted a different class of person from that assured in older days.

In Table XXVIII the mortality in England and Wales at various times is shown, and it will be noticed that there has been, on the whole, a general improvement, except at the oldest ages and in early infancy.

TABLE XXVIII.—CENSUS TABLES—RATE OF MORTALITY × 100

Table.	Period Investigated.	Age. 0.	5.	10.	15.	20.	30.	40.	50.	60.	70.	80.	90.
Male Lives—													
English Life No. 3	1838–1854	16·36	1·36	·56	·52	83	1·01	1·30	1·88	3·25	6·73	14·18	26·42
,, ,, ,, 4	1871–1880	15·86	·99	·40	39	·63	·94	1·39	2·01	3·54	6·99	14·48	28·28
,, ,, ,, 6	1891–1900	17·19	·71	·21	·30	·46	·67	1·19	1·94	3·60	7·21	15·20	29·19
,, ,, ,, 9	1920–1922	9·00	·42	·18	·22	·35	·43	·69	1·18	2·56	6·00	14·00	26·75
Eastern Counties Rural Districts	1920–1922	7·00	·27	·13	·18	·27	·37	·45	·72	1·64	4·31	12·36	...
Northumberland and Durham County Boroughs	1920–1922	11·47	·53	·23	·30	·50	·58	·95	1·50	3·33	7·92	17·49	...
German	1891–1900	23·39	·80	·30	·31	·58	·65	1·09	1·86	3·39	7·34	16·38	33·52
,,	1924–1926	11·58	·24	14	·19	·43	·41	·54	1·03	2·36	5·81	14·20	28·47
Swedish	1891–1900	11·08	·79	·39	·34	·65	·68	·82	1·26	2·26	5·12	13·13	30·68
,,	1921–1925	6·49	·23	·18	·24	·51	·45	·53	·88	1·84	4·24	11·22	26·60
Japanese	1899–1903	15·69	·79	·33	·47	·83	·79	1·04	1·77	3·51	7·44	15·58	32·72
,,	1921–1925	16·20	·70	·32	·60	1·08	·82	1·05	1·86	3·91	8·48	18·27	37·27
Female Lives—													
English Life No. 3	1838–1854	13·47	1·33	·58	·55	·86	1·06	1·28	1·62	2·88	6·06	13·02	24·83
,, ,, ,, 4	1871–1880	12·87	·91	·40	·40	·61	·86	1·16	1·59	2·92	6·14	13·10	25·81
,, ,, ,, 6	1891–1900	14·07	·71	·23	·31	·41	·62	1·00	1·50	2·93	6·24	13·63	26·38
,, ,, ,, 9	1920–1922	6·94	·42	·18	·23	·31	·39	·53	·92	1·90	4·65	11·77	23·85
Eastern Counties Rural Districts	1920–1922	5·22	·29	·13	·21	·34	·36	·43	·74	1·43	3·65	10·47	...
Northumberland and Durham County Boroughs	1920–1922	9·00	·53	·23	·29	·35	·52	·74	1·23	2·55	6·23	14·35	...
German	1891–1900	19·86	·81	·32	·35	·46	·70	·90	1·28	2·75	6·78	15·56	30·23
,,	1924–1926	9·39	·22	·12	·18	·33	·41	·53	·89	1·95	5·20	13·37	26·31
Swedish	1891–1900	9·21	·81	·40	·43	·53	·64	·77	1·01	1·80	4·41	11·79	26·87
,,	1921–1925	5·10	·23	·17	·27	·38	·42	·54	·83	1·58	3·80	10·13	25·60
Japanese	1899–1903	14·09	·81	·38	·64	·96	1·00	1·14	1·38	2·65	6·07	13·72	30·48
,,	1921–1925	14·40	·78	·37	·90	1·21	1·05	1·13	1·38	2·64	6·16	15·03	35·35
Both Sexes—													
Northampton	1735–1780	25·75	2·94	·92	·92	1·40	1·71	2·09	2·84	4·02	6·49	13·43	26·09
Carlisle	1779–1787	15·39	1·78	·45	·62	·71	1·01	1·30	1·34	3·35	5·16	12·17	26·06

Part of the change at the older ages is probably due to more accurate returns: there is evidence that people who are advanced in years are less likely to overstate their ages than they were in the days of the earlier censuses, and the members of the fair sex seem either to have developed better memories or to have learnt how to resist the temptation of giving themselves the benefit of a year or two when the census return is made. Apart, however, from these considerations, the mortality has changed, and several reasons have been advanced in explanation. One probable explanation is the improvement in methods of sanitation. It is also held that increasing knowledge, among the masses, of the proper treatment of children may have resulted in reduction of the mortality in childhood and youth, but may at the same time have led to the preservation up to middle age of people with feeble constitutions who would formerly have died young. The effect of the latter result would be to reduce the average vitality of those at the older ages, and might account for the variations in the rates of mortality at these ages.

It should be noticed that the change in mortality of men has been less than that of women, and that the lighter mortality in favour of men shown at some ages, when the earlier censuses were taken, has practically disappeared and their mortality is now heavier almost all through life. Some part of this change may be due to the improvement in the returns as regards age, to which we have referred.

It had been known for a long time that the mortality in towns was heavier than in the country, and when the English Life Table No. 9 had been made, tables were

also prepared for various sections of the country. Table XXVIII shows the heaviest and the lightest of these sectional mortality tables. It will be seen from these figures that the mortality of a country as a whole may give an incomplete impression of the mortality prevailing.

The following lines of the table show the results of a few of the investigations in countries outside the United Kingdom. In many countries there is a relatively heavy mortality, generally due to consumption, at early adult ages as compared with the mortality in childhood and after 30. This usually causes the mortality to increase rapidly, then keep level, and finally increase again when the ages are reached at which consumptive deaths become less important. In some countries, however, there may be found an actual decrease in the rates of mortality: in Norway, for instance, this can be seen at about age 30, and in Japan, where the adolescent stage is earlier than in Europe, the rate of mortality at 20 is higher than at 30.

The last two lines relate to two mortality tables (the Northampton and the Carlisle) which are nearly obsolete. The Northampton Table was constructed on unsound methods by Dr. Price in 1783 from the deaths in the Parish of All Saints, Northampton, during the years 1735–1780. Price's method would only have been accurate if the population had been stationary for a century before the date when the deaths he used were recorded. He was not blind to this source of error, and thought that the population was sufficiently stable, but the table showed too heavy a mortality, especially at the younger ages. It was in general use until better

tables replaced it, and in spite of the heavy mortality shown it was used by the Government in Pitt's time as the basis for the calculation of the annuities granted by the National Debt Office, the consequence being that the country must have lost heavily on such transactions. It is an example of the difficulty, we might almost say the impossibility, of constructing reliable tables of mortality from deaths or populations alone.

The Carlisle Table was based on the deaths for the years 1779–1787 and two censuses taken, January 1780 and December 1787, of two parishes in the City of Carlisle, and was constructed on sound lines by Joshua Milne in 1815. The particular district had a population which was fairly stationary, and in the term with which Milne was concerned there had been no epidemics nor other disturbing influences. The weakness in the table is that the ages were stated very inaccurately, but in spite of this it was for many years used for the calculation of premiums and the valuation of the liabilities of assurance companies, and it is still used to some extent for the valuation of reversions and in a few other special circumstances.

Table XXIX shows how the rates of mortality vary with the number of years since selection.

The first example is taken from the "20 Offices" Table (Healthy Males: $H^{[M]}$) which was constructed by a method similar to that described on p. 30. This table was formed from the combined experience of twenty British Offices and related only to cases accepted at ordinary rates of premium, which were on the books of the offices at any time before 1863. It was first published as an aggregate table,—or rather as two

aggregate tables, for the full aggregate and the table excluding the first five years of assurance were both given. Some years later Dr. T. B. Sprague made a further investigation of the original data and gave approximate values for the Select rates.

The next two examples are taken from the "British Offices Tables," which were constructed by the method of Chapter II, from the experience of sixty British Offices during the period 1863–1893. The $O^{[M]}$ Table gives the mortality under whole life with profit policies, and the $O^{[NM]}$ the mortality under whole life without profit policies. Tables for other classes of assurance were published and show very different rates of mortality, but they have not been used in practical work and it is unnecessary to reproduce them here.

The differences between the $H^{[M]}$ and $O^{[M]}$ Tables are very noticeable, but not more so than between the $O^{[M]}$ and $O^{[NM]}$.

The probable explanation of the latter result is that there was a certain amount of self-selection. A man who thinks he will live long is more likely to take a with profit policy than one who has doubts as to his prospects of reaching old age: it is also possible that the higher rate of premium is paid by a provident class, while many without profit policies are effected for purposes of loans among a class which we should not expect to be long lived. On the other hand, changing conditions may alter the position, and nowadays many without profit assurances are taken out by the most provident classes of the community to give as much immediate insurance as possible for dependants or to meet death duties.

The most recent experience of insurance mortality in Great Britain relates to 1924–1929, and was obtained by the census method on the lines of Chapter VII. Each contributing insurance office filled up schedules of the number of policies on its books on 1st January 1924, and on each subsequent 1st January up to and including 1st January 1930. Deaths were also returned. The schedules gave the number of policies in force and the number that had become claims by death, for each age in four classes: (1) Life, with profits; (2) life, without profits; (3) endowment assurances, with profits; (4) endowment assurances, without profits and separately for cases that had been accepted at the ordinary premium (*a*) after medical examination, (*b*) without medical examination. Each office might adopt any method it liked as regards age, and many different schemes were adopted owing to the different ways in which offices kept their books. The returns of the offices had to be adjusted to a common basis, and after adjustment the figures gave the "populations" at nearest age and the deaths at nearest age and, consequently, the method described in Chapter VII (see p. 77) gave the rates of mortality at each half age or, which is approximately the same thing, at each age last birthday.

The statistics were subdivided by the offices according to durations 0, 1, 2, 3, 4, and 5 or over, *i.e.* six groups in all, and rates of mortality were therefore obtained for these durations.

A mortality table was prepared for use, combining all classes, medical and non-medical and, for convenience, this final table analysed the rates of mortality

only according to durations 0, 1, and 2, followed by an ultimate table for durations 3 and over. The table is called A 1924–29, and shows a much lighter mortality than that of the earlier experiences. To the extent that it includes the lighter mortality of endowment assurances, it is not strictly comparable with the $O^{[M]}$ experience, which related only to whole life assurances.

It is interesting that in the six years 1924–1929 the without profit policies showed a rather lighter mortality than with profit policies, and that there was little difference in the mortality of the medical and non-medical business. The statistics are still being examined, and further information for subsequent years is being collected continuously.

The remaining examples relating to assured lives have been selected from the well-known tables published in other countries. The extent of the observations in each case is indicated in the table, and the methods of construction were similar to that employed for the $O^{[M]}$ Tables. The reader will be able to draw his own conclusions from a comparison of these figures with those for the English Tables, but some allowance must be made for chronological differences. In connection with the Japanese Offices experience, it may be mentioned that the extra mortality due to the war with Russia was excluded in arriving at the figures given.

The second part of the table shows the results of Annuity Experiences. Here the selection is effected entirely by the annuitant. Obviously, a man or woman does not buy an annuity if he or she thinks there is

TABLE XXIX.—LIFE OFFICE TABLES—SELECT RATES OF MORTALITY × 100

Table.	Period investigated.	Age at Entry.								
		25.			40.			55.		
		1st Year of Assurance.	2nd Year of Assurance.	6th Year of Assurance.	1st Year of Assurance.	2nd Year of Assurance.	6th Year of Assurance.	1st Year of Assurance.	2nd Year of Assurance.	6th Year of Assurance.
Male Lives—										
20 Offices: Institute of Actuaries H[M]	Up to 1863	·45	·74	·92	·55	·84	1·29	1·04	1·65	3·06
British Offices: with profit whole life policies O[M]	1863–1893	·28	·46	·65	·44	·66	1·04	1·04	1·42	2·51
British Offices: without profit whole life policies O[NM]	1863–1893	·45	·61	·83	·65	·85	1·30	1·44	1·79	3·11
British Offices: whole life and endowment assurances A 1924–29	1924–1929	·16	·20	·24	·24	·34	·53	·69	1·04	1·97
Gotha Life Office—Germany	1852–1896	·32	·41	·54	·48	·65	1·03	1·08	1·63	2·80
28 Austrian Offices	1876–1900	·36	·61	·90	·62	1·01	1·42	1·50	2·40	3·18
18 Hungarian Offices	1876–1900	·51	·82	·96	·78	1·17	1·64	1·67	2·36	3·86
3 Japanese Offices	Up to 1905	·64	·83	·82	·62	·93	1·48	1·86	2·82	4·49
Japanese Offices J[PM]	1912–1927	·55	·90	·80	·74	1·02	1·31	2·39	2·96	4·14

Table.	Period investigated.	Age at Date of Purchase.								
		50.			65.			80.		
		1st Year of Contract.	2nd Year of Contract.	5th Year of Contract.	1st Year of Contract.	2nd Year of Contract.	5th Year of Contract.	1st Year of Contract.	2nd Year of Contract.	5th Year of Contract.
Male Lives—										
Government Annuities . .	1808–1875	1·37	2·03	2·42	2·93	3·31	5·96	6·67	11·26	18·63
,, ,, . .	1875–1904	·82	1·24	2·00	2·28	3·26	5·47	7·50	10·40	17·33
,, ,, . .	1900–1920	·8	1·07	1·30	2·0	3·64	4·79	7·6	13·77	17·74
British Offices : Annuitants O[am]	1863–1893	·82	1·08	1·90	2·44	3·19	5·41	8·22	10·64	17·38
,, ,, ,, a(m)	1900–1920	·61	1·04	1·31	1·92	3·29	4·14	6·87	11·77	14·75
,, ,, ,, experience	1900–1920	·7	1·0	1·4	1·9	2·9	5·1	7·0	10·3	15·9
Female Lives—										
Government Annuitants . .	1808–1875	·78	·95	1·75	1·75	2·23	4·34	6·55	9·33	16·52
,, ,, . .	1875–1904	·48	·71	1·47	1·35	2·00	3·92	5·25	7·67	14·39
,, ,, . .	1900–1920	·5	·88	1·00	1·3	2·13	2·91	5·5	10·21	13·75
British Offices : Annuitants O[af]	1863–1893	·61	·94	1·71	1·39	2·06	3·92	6·57	8·80	15·02
,, ,, ,, a(f)	1900–1920	·53	·87	·96	1·14	1·96	2·53	4·80	8·51	11·90
,, ,, ,, experience	1900–1920	·6	·8	1·0	1·2	1·8	3·2	4·6	6·9	13·0

much chance of early death, and the effect of this self-selection is remarkable.

The first two examples of both male and female annuitant mortality are from the experience of persons purchasing annuities from the National Debt Office; the first experience extending from 1808–1875[1] and the second from 1875–1904.

The third example relates to the combined experience of the National Debt Office and the Post Office; the published mortality table gives only an aggregate table excluding the first year after purchase, and these rates of mortality appear in our table with approximate figures for the first year of the contract.

Investigations into the mortality of annuitants were made from the experience of British insurance companies for the periods 1863–1893 and 1900–1920. The methods were similar to those of the British Offices 1863–1893 assurance experience, except that for 1900–1920 all the work was done for ages last birthday at entry, and the rates of mortality obtained were subsequently adjusted to show the mortality at exact ages. The tables made from the 1863–1893 experience gave select rates of mortality for five years and then an ultimate table, but the 1900–1920 experience was used to make an estimate of the mortality that would subsequently be found to exist for entrants in 1925, and gave only one year of selection. In order to facilitate comparison we have therefore also given approximate figures for the rates of mortality experienced during the period 1900–1920. It will be noticed that the mortality

[1] The method of construction of this table was different from that of the other tables discussed.

of female annuitants has improved more than that of male annuitants.

Table XXX contains examples from Aggregate Tables of Mortality. It is possible for a comparison of such tables to give results which do not seem consistent with those given by a comparison of the corresponding Select Tables, because the proportion of recently selected lives included in the experiences may vary (see p. 91). The figures given are, however, interesting, and it is useful to compare the Aggregate Tables with the rates found from the Census returns (Table XXVIII).

A rather more useful comparison may be made between the rates of mortality of the general community and those given by aggregate tables, excluding the years of insurance when selection is most important (see Chapter III, page 24). Table XXXI gives such a comparison, but it is again necessary to notice chronological differences in making use of this Table.

In Table XXX are included the rates of mortality found from a few friendly society experiences. These societies have, in the past, consisted mainly of members of the superior artisan and labouring classes, who joined their friendly society more or less as a matter of course on attaining manhood. In consequence we do not find so much "selection" of the kind already mentioned as that observed in the experience of life insurance offices, but if the rates of mortality are compared with those of the general population it will be seen that there is a "selection" of another kind, due to the fact that these figures represent the

TABLE XXX.—AGGREGATE RATES OF MORTALITY—RATE OF MORTALITY × 100
MALE LIVES

Table.	Period investigated.	Age attained.				
		20.	30.	40.	50.	60.
Life Assurance Offices—						
20 Offices	Up to 1863	·63	·77	1·03	1·60	2·97
British Offices—with profit policies	1863–1893	·40	·59	·92	1·50	2·89
Austrian Offices	1876–1900	·56	·73	1·10	1·93	3·72
Hungarian Offices	1876–1900	·70	·90	1·30	2·13	3·83
Japanese Offices	Up to 1905	·89	·72	·94	2·03	4·37
" "	1912–1927	·98	·73	·95	1·86	4·13
Friendly Societies—						
Sutton's Registered Friendly Societies	1876–1880	·70	·71	1·07	1·73	3·40
Watson's Manchester Unity—						
Non-manufacturing localities	1893–1897	·28	·50	·77	1·30	2·62
Textile industries localities	1893–1897	·34	·52	1·02	1·77	3·88
Manufacturing localities	1893–1897	·35	·52	·84	1·48	3·10

TABLE XXXI.—AGGREGATE RATES OF MORTALITY (EXCLUDING THE EXPERIENCE OF THE FIRST FEW YEARS OF INSURANCE) × 100 COMPARED WITH RATES OF MORTALITY SHOWN BY ENGLISH LIFE NO. 6 TABLE—MALE LIVES

Table.	Period investigated.	Number of Years excluded.	Age attained.				
			30.	40.	50.	60.	70.
English Life No. 6 . .	1891–1900	..	·67	1·19	1·94	3·60	6·24
20 Offices	Up to 1863	5	·92	1·13	1·71	3·06	6·28
British Offices—with profits .	1863–1893	5	·75	·98	1·54	2·92	6·22
British Offices—without profits	1863–1893	5	·83	1·07	1·66	3·11	6·56
British Offices . . .	1924–1929	3	·24	·39	·76	1·97	5·33
Government Annuitants .	1808–1875	4	1·24	1·64	2·16	2·85	6·46
Government Annuitants .	1900–1920	1	·47	·66	1·01	2·09	5·24
British Offices Annuitants .	1863–1893	5	·76	·99	1·54	2·85	5·93
British Offices Annuitants .	1900–1920	1	..	·80	·97	2·08	4·46

experience of the more prosperous and provident portion of the community.

Table XXXII contains examples of the rates of sickness that have been experienced by some of the oldest friendly societies, and in connection with National Health Insurance. Here, as elsewhere, different methods influence the results to some extent, and too much stress must not be laid on small differences. It is, however, interesting to notice that the rates of sickness have increased from time to time, whereas the rates of mortality among the same people have decreased.

The tables examined were formed from the experience of the Independent Order of Oddfellows (Manchester Unity) and the returns made to the Registrar of Friendly Societies by all registered English Friendly Societies during 1876–1880.

The second Manchester Unity Experience here dealt with (1893–1897) was extensively analysed. The rates of mortality given in Table XXX will be seen to relate to three different areas of the country, and the sickness rates in Table XXXII are those found in two of the groups of occupations. This experience is particularly interesting, as the "Whole Society" rates formed the sickness basis of the calculations under the National Health Insurance Acts. Those Acts related to women as well as men, and Table XXXII gives rates of sickness for both sexes. The earlier Friendly Societies statistics related only to men.

Table XXXIII is added to show how rates of withdrawal vary with the age and period of membership. The figures, of course, depend almost entirely on the

TABLE XXXII.—RATES OF SICKNESS

Experience.	Period investigated.	Age at time of Illness. 30.	40.	50.	60.
		First six months or "sickness benefit" under National Health Insurance.			
Manchester Unity of Oddfellows	1866–1870	·74	·89	1·28	2 11
Registered Friendly Societies (W. Sutton)	1876–1880	·78	1·00	1 40	2·12
Manchester Unity (A.W.Watson)—					
Agricultural and Miscellaneous (A.H.J.)	1893–1897	·74	·91	1·24	1·92
Mining Occupations (G.)	1893–1897	1 42	1 73	2·31	3·42
Whole Society	1893–1897	·82	1·00	1 34	2·05
Whole Society, adjusted to compare with N.H.I.	1893–1897	·72	·90	1·24	1·94
National Health Insurance—					
Men	1921	·58	·61	·76	1·45
Men	1927	·76	·90	1·16	1·88
Unmarried Women	1921	·69	·75	·95	1·30
Unmarried Women	1927	1·11	1 20	1·50	1·98
Married Women	1921	1·22	1·22	1·36	1·92
Married Women	1927	2·86	2·49	2·60	2·66
		All periods combined or "sickness" and "disablement" benefits.			
Manchester Unity of Oddfellows	1866–1870	86	1·15	1·96	3·98
Registered Friendly Societies	1876–1880	·96	1·37	2·18	4·32
Manchester Unity (A.H.J.)	1893–1897	·92	1·31	2·18	4·82
Manchester Unity (G.)	1893–1897	1·71	2·46	4·44	9·32
Manchester Unity, Whole Society	1893–1897	1·01	1·45	2·38	5·20
Manchester Unity, Whole Society adjusted to compare with N.H.I.	1893–1897	·91	1·34	2·33	5·28
National Health Insurance—					
Men	1921	·76	·95	1·34	3·23
Men	1927	1·14	1·58	2·27	4·95
Unmarried Women	1921	1·21	1·42	1·88	3·44
Unmarried Women	1927	2·10	2·68	3·65	6·39
Married Women	1921	1·61	1·92	2·73	4·85
Married Women	1927	4·11	4·37	5·84	8 64

Note.—The National Health Insurance rates and the adjusted rates for the Manchester Unity are obtained approximately from figures given by Sir Alfred W. Watson in the *Journal of the Institute of Actuaries* in 1931.

TABLE XXXIII.—RATES OF SECESSION PER CENT. PER ANNUM IN MANCHESTER UNITY FRIENDLY SOCIETY DURING 1893–1897

(Extract from a table given by Sir A. W. Watson in his introduction to the account of the investigation of the experience of the Society)

Ages.	Rural Districts.			Urban Districts.			Rural and Urban Districts Combined.		
	Whole Experience.	Experience after First 6 Calendar Years of Membership.	Experience after First 11 Calendar Years of Membership.	Whole Experience.	Experience after First 6 Calendar Years of Membership.	Experience after First 11 Calendar Years of Membership.	Whole Experience.	Experience after First 6 Calendar Years of Membership.	Experience after First 11 Calendar Years of Membership.
20–24	5·02	3·78	...	7·96	4·51	...	6·29	4·06	...
30–34	3·21	2·35	1·98	5·14	3·15	2·45	4·11	2·71	2·17
40–44	1·50	1·16	1·05	2·14	1·52	1·28	1·79	1·32	1·15
50–54	0·57	0·56	0·54	0·82	0·81	0·77	0·68	0·68	0·64
60–64	0·34	0·34	0·35	0·45	0·45	0·45	0·39	0·39	0·39

voluntary action of the members themselves, but it is interesting to notice how comparatively regular such action is, the general principle shown being, as one would expect, that those who have most to lose are least inclined to cancel their contracts. The rates of withdrawal found from the experience of any society are peculiar to that society, and if employed in any calculations they must be used with great caution, and care must be taken to see that the conditions of the society have not changed so as to affect them in any way. The figures from one experience can seldom, if ever, be used in making investigations in connection with another society.

The reader will probably have noticed that it is sometimes a little difficult from a study of tables like those given in this chapter to form a general idea of the differences between the mortality shown by various experiences. The difficulty is greater when we are dealing with the unadjusted rates of mortality, *i.e.* before a graduation has been made. To overcome this inconvenience and to facilitate comparisons other functions are sometimes employed by actuaries. We have already mentioned the method of comparing the actual deaths with those expected by a standard table, and we can extend its usefulness by making the comparison between the totals of the figures in groups of ages and for all ages. A method used by the Registrar-General of Births, Deaths, and Marriages is to show how many persons remain alive at each age in various experiences out of 1,000,000 births, and another method in frequent use is to compare the "expectations of life" shown by different tables. The last function

TABLE XXXIV.—VALUES OF TEMPORARY ANNUITIES BY VARIOUS TABLES—INTEREST, 3 PER CENT.

Table.	Age.	Term of Annuity.			
		10.	15.	20.	25.
English Life No. 3 .	20	8·146	11·163	13·622	15·609
	30	8·062	10·988	13·322	15·151
	40	7·912	10·662	12·757	14·287
	50	7·596	9·995	..	..
20 Offices, aggregate.	20	8·236	11·345	13·910	16·012
	30	8·165	11·189	13·641	15·592
	40	8·037	10·901	13·122	14·768
	50	7·731	10·234	11·964	13·042
British Offices, life, with profits, select.	20	8·312	11·460	14·061	16·196
	30	8·271	11·356	13·860	15·857
	40	8·171	11·109	13·390	15·086
	50	7·935	10·541	12·355	13·494
British Offices, same, aggregate . .	20	8·332	11·520	14·171	16·354
	30	8·232	11·311	13·816	15·819
	40	8·072	10·967	13·218	14·894
	50	7·759	10·286	12·044	13·145
British Offices, same, excluding first five years . . .	20	8·234	11·343	13·915	16·026
	30	8·180	11·221	13·689	15·658
	40	8·050	10·927	13·160	14·820
	50	7·743	10·258	12·005	13·099
A. 1924–29 . .	20	8·436	11·744	14·556	16·936
	30	8·422	11·696	14·446	16·728
	40	8·342	11·502	14·067	16·069
	50	8·131	10·975	13·057	14·424
Carlisle . . .	20	8·214	11·278	13·786	15·815
	30	8·083	11·026	13·386	15·279
	40	7·922	10·750	12·973	14·610
	50	7·834	10·374	12·149	13·299

is simply the average number of years lived by persons of any age after the attainment of that age, *i.e.* the average future lifetime. Against the use of this function the objection may be made that although it may give a good general idea of the mortality shown throughout a particular table, it disguises even appreciable differences in the rates of mortality if they tend to counterbalance one another; so that it is possible for the expectation of life at a certain age to be the same in two tables which show rates of mortality differing widely throughout. A better comparison could be made by the use of "temporary expectations of life," that is to say, the average duration of life during a few years only from the attainment of any age. Such values are not often calculated, but an exactly similar function is the value of a temporary annuity, payable until the expiration of a fixed number of years and subject to earlier discontinuance if the annuitant should die. Table XXXIV has therefore been added, and contains examples of the values of such annuities at a few ages and for specified terms, on the basis of some of the Mortality Tables discussed in this chapter. The reader should examine the figures given and see how the conclusions arrived at compare with those obtained from a study of the earlier tables.

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